DETAILED SYLLABUS

TWO YEAR (FOUR SEMESTERS) M.SC. DEGREE COURSE (AUTONOMOUS)
IN MATHEMATICS,
BETHUNE COLLEGE, KOLKATA
2015-2016.

GENERAL PAPERS

MODULE 101 (GROUP: A): ABSTRACT ALGEBRA- I (25 Marks)

Group Theory

1. **Recapitulation** : Subgroup, permutation group, Normal subgroup , quotient group, Homomorphism, Isomorphism Theorem.
2. **Group Action on sets:** Cayley's theorem, Conjugacy class, class equation, Cauchy's theorem on finite groups & p-groups, Sylow's Theorem & application.
3. **Simple and non-simple groups:** Simple group & their characterization, identification of all simple group of order 60, Non simplicity of group. Classification of all groups up to order 8.

Ring Theory

4. Ideal, Quotient ring, Isomorphism theorems, prime & maximal ideals.

References:

1. D. S. Malik, J. H. Mordeson and M. K. Sen; **Fundamentals of Abstract Algebra**, McGraw-Hill, 1997.
2. T. H. Hungerford; **Algebra**, Holt, Reinhart and Winston, 1974.
3. I. N. Herstein; **Topics in Algebra**, Wiley Eastern Ltd. New Delhi, 1975.
4. Joseph J. Rotman; **An Introduction to the Theory of Groups**, Springer-Verlag, 1990.
5. S. Lang; **Algebra** (2nd ed.), Addition-Wesley.
6. D. S. Dummit, R. M. Foote; **Abstract Algebra**, 2nd edition, Wiley Student Edition.
7. Michael Artin; **Algebra**, PHI (Eastern Economy Edition).

MODULE 101 (GROUP: B): LINEAR ALGEBRA (25 Marks)

1. **Recapitulation:** Vector Spaces, sub-spaces, generator, characterization of generating set, linear independence, linear dependence, basis of a finite dimensional vector space, eigen values, eigen vectors.
2. **Linear Transformation:** The three isomorphism theorems, matrix representations of linear transformations.
3. **Infinite dimensional vector spaces:** Infinite dimensional vector spaces, existence of basis, equality of cardinality of two bases, some characterizations.
4. **Dual Space:** Dual space, double dual, Natural isomorphism between dual and double dual spaces, dual basis.
5. **Inner Product Spaces:** Inner product, Norm, IPS, orthonormal bases, orthonormal projection, Gram-Schmidt process of orthogonalisation.
6. **Diagonalisation of Linear Transformations and Canonical Forms:** Characteristic polynomials, eigen values, eigen vectors of linear transformation, annihilating polynomials, minimal polynomials, invariant sub-spaces, simultaneous triangulation and Diagonalisation. Direct sum decomposition and projection, invariant direct sum. Primary decomposition theorem. Rational canonical forms. Cyclic sub-spaces, nil-cyclic sub-spaces, cyclic decompositions, Jordan form.

References

1. S. Axler, **Linear Algebra Done Right**, Springer, New York, 1996.
2. T. S. Blyth, **Module Theory**.
3. W. Cheney & D. Kincaid, **Linear Algebra,** 1st ed., Jones & Bartlett, 2010.
4. S. H. Friedberg, A. J. Insel, L. E Spence, **Linear Algebra**, 4th ed., Prentice Hall, 2002.
5. P.R. Halmos, **Finite-Dimensional Vector Spaces**, 2nd ed., Springer, 1987.
6. I.N. Herstein, **Algebra,** 2nd ed., John Wiley & Co., 1975.
7. K. Hoffman & R. Kunze, **Linear Algebra**, 2nd d., Prentice Hall Inc., 1971.
8. S. Roman, **Advanced Linear Algebra**, 3rd ed., Springer, 2008.

MODULE 102: REAL ANALYSIS – I (50 Marks)

1. **Cardinal Numbers:** Concept of cardinal number of an infinite set, order relation in cardinal numbers, Schroder-Bernstein theorem, arithmetic operations on cardinal numbers, Axiom of choice, continuum Hypothesis.
2. **Functions in $\mathbb{R}$**: Weierstrass Approximation theorem, weierstrass non-differentiable function, continuity and differentiability of convex function.
3. **Functions in $\mathbb{R}^n$** : Concept of continuity and differentiability for functions from $\mathbb{R}^m$ to $\mathbb{R}^n$ and their simple properties, partial and directional derivatives, total derivative and its expression in terms of partial derivatives, sufficient condition for differentiability, mean value theorem for differentiable functions.
4. **Lebesgue Measure:** Lebesgue outer measure, Lebesgue measurable sets, Borel sets in $\mathbb{R}$, Approximation of Lebesgue measurable sets from inside or outside by simple sets, Non- measurable sets.
5. **Lebesgue Measurable functions:** Definitions and basic properties, algebraic operations on measurable functions, sequence of measurable functions, Egoroff's theorem, simple functions, approximation of measurable function by simple functions, convergence in measure and almost everywhere convergence with their interrelation.

References:

1. T. M. Apostol, **Mathematical Analysis**, Narosa Publishing House
2. J. F. Randolph, **Basic Real and Abstract Analysis**, Academic Press, N.Y.
3. W. Rudin, **Principles of Mathematical Analysis**, MC Graw-Hill, N.Y.
4. W. Sierpinski, **Cardinal and Ordinal Numbers**.

MODULE 103: COMPLEX ANALYSIS (50 Marks)

1. Complex integration : Curves, Jordan Curve theorem (statement only), complex integration along a curve, Basic properties, winding number of a closed curve, Cauchy-Goursat theorem, Cauchy's integral formula, Cauchy's integral formula for higher derivatives, Cauchy's inequality, Morera's theorem, Liouville's theorem on entire functions, Fundamental theorem of algebra.
2. Uniform convergence and power series : Uniform convergence of a sequence and series of functions, Weierstrass' M-test, Weierstrass theorem on uniformly convergent sequence and series of analytic functions. Power series, Cauchy-Hadamard theorem, uniqueness theorem for Power series.
3. Many-valued functions : Branches of many-valued functions with Special reference to $Arg z, Log z \text{ and } z^{\alpha}$. Branch points.
4. Analytic functions : Taylor's theorem, zeros of an analytic function, Interior Uniqueness theorem for analytic functions, Maximum-modulus and minimum-modulus principles, Schwarz' lemma.

5. Laurent Series : Laurent's theorem, classification of singularities pole, Essential singularity, Removable singularity, Behaviour of a function near a pole, Riemann's theorem on removable singularity,Casorati-Weierstrass theorem.
6. Theory of residues and Meromorphic functions : Residue, Cauchy's residue theorem, Contour integration, Meromorphic functions, Argument principle, Rouche's theorem, Deduction of fundamental theorem of algebra.
7. Conformal mapping and Bilinear Transformation : Conformal mapping, Basic properties, Bilinear Transformation (B.T.) , Cross-ratio, Cross-ratio preserving property of a B.T, Fixed points of a B.T, B.T as a composition of translation, dilation and inversion, Uniqueness of a B.T by its effect on three distinct points in $\mathbb{C}_\infty$.

References:

1. A. I. Markushe-Vich, **Theory of Functions of a Complex Variable – Vol I & II,** Prentice-Hall.
2. S. Ponnusamy, **Foundations of Complex Analysis** (2nd Edition), Narosa Publishing House.
3. J.B.Conway, **Functions of One Complex Variable,** Narosa Publishing House.
4. R.V. Churchill and J.W. Brown, **Complex Variables and Applications,** Mcgraw-Hill.
5. L.V. Ahlfors, **Complex Analysis,** Mc Graw-Hill.

MODULE 104 (GROUP: A): FUNCTIONAL ANALYSIS- I (25 Marks)

Banach Contraction Principle Theorem in a complete metric space and its application as Picards Theorem in O.D.E and as Implicit function theorem. Bounded set and Diameter of a set in metric space (X, d); Distance function of a point from a set in (X, d) is a continuous function; separation of disjoint pair of closed sets in (X, d) by continuous function. Compact metric space, compact sets in (X, d) being bound and closed; total boundedness of a set and €-nets in (X, d); Arzela Ascoli Theorem.

Normed Linear spaces (NLS) –Banach space –example of Banach spaces like that of $\mathbb{R}^n$, C[a, b], sequence space $l_p(1 \leq p < \infty)$ Quotient space of a NLS; continuity of a Norm function. Bounded Linear operator over NLS, its Continuity at a point of NLS. Joining its continuity over space; Boundedness and continuity of a Linear operator, Unbound Linear operator. Norm of a bounded linear operator $T = \|T\|$, formulation of $\|T\|$ for its estimate. Equivalent norms; Reisz Lemma and finite Dimensionality of NLS by compact unit ball, Boundedness (Continuity) of Linear operator over NLS $(X, \|T\|)$ with Dim (X) < ∞.

References :

1. Leisternik and Soleolev; **Introduction to Functional Analysis**
2. B. K. Lahiri; **Elements of Functional Analysis**
3. B. V. Limaye; **Elements of Functional Analysis**
4. Bachman and Narici; **Functional Analysis**
5. Kreyszig; **Introduction to Functional Analysis**

MODULE 104 (GROUP: B): TOPOLOGY- I (25 Marks)

Topological spaces, Definition and examples, Comparison of Topologies, Indiscrete and discrete Topology, Intersection and Union of Topologies; neighbourhood of a point, neighbourhood system at a point – properties of neighbourhood, Limit point of a set, closure of a set; Interior, exterior and boundary of a set and their mutual relations. Bases and sub-bases of a Topology; Base for usual Topology, upper limit Topology and lower limit Topology of reals. First and Second countable spaces, Separable spaces.

Kuratowski closure operator and generating Topology; Product Topology and Product space, Projection functions and their properties; Quotient space. Convergence in Topological spaces; Nets and filters – their properties. Sub-spaces and relative Topology; Continuous function and its various characterizations; Homeomorphism; Topological invariants.

References :

1. J. L. Kelley; **General Topology**
2. J. Dugundji; **Topology**
3. Munkres; **Topology**
4. W. J. Thron; **Topological Structures**
5. G. F. Simmons; **Introduction to Topology and Modern Analysis.**

MODULE 105 (GROUP: A): CLASSICAL MECHANICS- I (25 Marks)

1. **Calculus of variations**: Introduction, Euler's equation, Fundamental problem and its solution, cases of several dependent and independent variables of first and higher derivatives. Isoperimetric problem and its solution. Applications.
2. **Classical Mechanics:** Mechanics of a system of particles, constraints, Generalized co-ordinates, Virtual displacement and principle of V.W.
3. D'Alembert's principle, Generalized forces, Lagrangian, Lagrange's equations of motion, Velocity dependent potential, Properties of K.E. function, Conservation of energy, Rayleigh dissipation function. Cyclic co-ordinates, symmetry properties and conservation laws.
4. Legendre transformation, Hamiltonian, Hamilton's equations, Hamilton's principle.

References:

1. H. Goldstein; **Classical Mechanics** (Addison Wesley), 1950
2. A. Sommerfeld; **Mechanics, Academic Press**, N.Y. 1952
3. E. T. Whittaker; **Analytical Dynamics** (Cambridge University Press)
4. F. Gantmacher; **Lectures on Analytical Mechanics** (MIR Publishers),1975
5. N.C. Rana and P.S. Joag; **Classical Mechanics**, Tata McGraw Hill, New Delhi, 1998
6. A.K. Raychaudhuri
7. P.V. Panat

MODULE 105 (GROUP: B): DISCRETE MATHEMATICS- I (25 Marks)

1. Lattice Theory : Partially Ordered Set (Poset), Lattice as Poset, Lattice as algebraic structure, Sublattice, Direct Product, Homomorphism of lattices, Congruences, Complemented lattice, Modular lattice, Distributive lattice, Boolean Algebra, Atom and Join -irreducible elements,Boolean Functions, Conjunctive and Disjunctive Normal forms, Stone representation theorem for Boolean Algebra.
2. Logic : Propositional Logic, Language,Well-formed Formulae, Truth table, Tautology, Contradiction, Basic Theorems, Connection of Propositional Logic with Boolean Algebra, Axiomatic approach to Propositional Logic, Syntax, Deduction Theorem, Lindenaum Algebra, Semantics, Soundness, Consistency of Axiomatic system.
3. Formal language and Automata: Finite State Machines, Moore and Mealy machines.

References:

1. E. Mendelson; **Introduction to Logic**, AP
2. Margaris; **First order Mathematical Logic**, Blaisdell Publishing Co.
3. D.S. Malik and M.K. Sen; **Discrete Mathematical Structures, Theory and Applications**, Thompson.

4. J.E. Hopcroft and J.D.Ullman; **Introduction to Automata Theory, Languages and Computation**.
5. H.R.Lewis and C.H.Papadimitriou; **Elements of the Theory of Computation.**

MODULE 201 (GROUP: A): ABSTRACT ALGEBRA-II (25 Marks)

Ring Theory

1. Polynomial ring & factorization of polynomials over a commutative ring with identity, PID & UFD.
2. Definition & examples of Euclidean Domains, Eisenstein's theorem.
3. Chinese Remainder theorem, primitive roots.

Field Theory

4. Fields Extensions(algebraic and transcendental), roots of polynomials, splitting field of a polynomial, finite fields.
5. Galois Theory.

References:

1. Malik, Mordeson and Sen; **Fundamentals of Abstract Algebra**; McGraw-Hill, 1997.
2. T. H. Hungerford; **Algebra**; Holt, Reinhart and Winston. 1974.
3. I. N. Herstein; **Topics in Algebra**; Wiley Eastern Ltd. New Delhi, 1975.
4. Joseph J. Rotman; **An Introduction to the Theory of Groups**; Springer-Verlag, 1990.
5. S. Lang; **Algebra** (2nd ed.); Addition-Wesley.
6. D. S. Dummit, R. M. Foote; **Abstract Algebra**, 2nd edition; Wiley Student edition.
7. Michael Artin; **Algebra**; PHI. (Eastern Economy Edition) Prentice Hall.

MODULE 201 (GROUP: B): DIFFERENTIAL GEOMETRY (25 Marks)

Preliminaries of Tensor calculus, Intrinsic derivative, Parallel vector fields, Theory of space curves, Theory of surfaces, Curves on surfaces, First and second fundamental forms, Geodesic and Geodesic curvature, Isometric surfaces, Gaussain curvatures, Condition of integrability, Gauss's formula, Gauss-Codazzi-Weingarter equations, Third fundamental form, Meusnier's theorem, Principal directions and principal curvatures, Asymptotic directions.

References

1. I. S. Sokolnicoff ; **Tensor Calculus and Applications to Geometry and Mechanics.**

MODULE 202 : REAL ANALYSIS- II (50 Marks)

1. **Metric Spaces:** Metric Space Revisited, Equivalent metrices, Complete metric spaces and subspace of complete metric space, Completion of metric spaces, Baire Category theorem, Banach contraction principle and its applications, compactness, equivalence of compactness and sequentially compactness, total boundedness, separability of totally bounded set, characterization of compactness of totally boundedness, concept of equicontinuity and Arzela-Ascoli theorem, continuity and uniform continuity of a function from a metric space into another metric space, Extension theorem.
2. **Riemann-Stieltjes Integral:** Definition and existence of Riemann-Stieltjes Integral, Some properties, Integration by-parts, Riemann-Stieltjes Integral as a Riemann Integral.
3. **Lebesgue Integration:** Integration of simple functions, Lebesgue integral of a bounded function and its characterization, comparision of Rieman integral and Lebesgue integral, Integration of non-

negative valued measurable functions and some simple properties, Fatou's lemma, Monotone convergence theorem, Integrable functions, Dominated convergence theorem.

References:

1. A.M. Bruckner, J. Bruckner and B, Thomson; **Real Analysis**, Prentice-Hall N.Y.
2. P.K. Jain and K.Ahmad; **Metric Spaces**, Narosa Publishing, N.Y.
3. G.D. Bana; **Measure Theory and Integration**, Wiley Eastern Limited, 1987
4. I.K. Rana; **An Introduction to Measure and Integration**, Narosa Publishing House, 1997.
5. P.K. Jain and V.K. Gupte; **Lebesgue Measure and Integration**, New Age International(P) Limited Publishing Co., New Delhi.

MODULE 203 (GROUP: A): FUNCTIONAL ANALYSIS- II (25 Marks)

Space BdL (X,Y) of all bdd. Linear `operators from NLS (X, ǁ ǁ) to (Y, ǁ ǁ). Its completeness; Linear functional over (X, ǁ ǁ), sub-Linear functional over (X, ǁ ǁ), Hahn-Banach Theorem, and its applications. First and Second dual spaces of (X, ǁ ǁ). Canonical mapping, Enebedding(X, ǁ ǁ) into its Second Dual under Linear Isometry. Reflexive Banach space, open mapping theorem and closed graph theorem. I. P. space; C-S Inequality and I. P. space as NLS. Continuity of I. P. function. Law of parallelogram; orthogonal (orthogonal system and Linear independence).

Hilbert space H; its examples. Projection Theorem in H, Ring representation Theorem for a bdd. Linear functional over H, Self-dual property of H. Bessels Inequality – complete orthogonal system in H and Persevals equalty. Compact Linear operator over NLS, Bddness of compact Linear operator; limit of a sequence compact Linear operators, Fredholm Integral operator asa compact operator; Hilbert adjoint and self-adjoint operators in H. Bdd Linear operator over H as a sum of self-adjoint operators; eigen values and eigen vectors of a self adjoint operator.

References :

1. Leisternik and Soleolev; **Introduction to Functional Analysis.**
2. Lahiri; **Elements of Functional Analysis.**
3. Limaye; **Elements of Functional Analysis.**
4. Bachman and Narici; **Functional Analysis.**
5. Kreyszig; **Introduction to Functional Analysis.**

MODULE 203 (GROUP: B): TOPOLOGY- II (25 Marks)

Separation axioms: T_0, T_1, T_2 and their characterizations in a Topological spaces. Regular space, Complete regular space and T_3 – space, Normal space and T_4 – space. Urysohn's Lemma, Tietze Extension Theorem on continuous function; Open cover and compactness in Topological spaces. Compact space in Topological spaces. Heive-Borel Theorem in $\mathbb{R}^n$- space; continuous image of compact space is compact Finite intersection property, Tychonoff Theorem on product of compact space, Local compactness.

Separated sets in Topological spaces, Connectedness, connected sets of reals in usual Topology, continuous image of connected space is connected components and their properties; Local connectedness. Uniform space, Uniform Topology; Hausdorff property in Uniform space, Base for a Uniformity; Uniform Continuity and Continuity of a function over Uniform Space.

References :

1. J. L. Kelley; **General Topology**
2. J. Dugundji; **Topology**
3. Munkres; **Topology**

4. W. J.Thron; **Topological Structures**
5. G. F. Simmons; **Introduction to Topology and Modern Analysis.**

MODULE 204: ORDINARY DIFFERENTIAL EQUATIONS & SPECIAL FUNCTIONS (50 Marks)

1. Linear ordinary differential equation, Initial value problem, Existence and Uniqueness of Solutions, Wronskian, Linear Independence of solutions, Equations in matrix form, Fundamental system of solutions, adjoint and self adjoint, General solution of inhomogeneous equation.
2. Frobenius method, Series solution of linear second order ordinary differential equation, Indicial equation, Regular singularity, Equation of Fuchsian type.
3. Hermite's equation, Hypergeometric equation, Legendre equation, Bessel equation, Strum-Liouville boundary value problem, Eigen values and Eigen functions, Self adjoint problem, Properties of Eigen values and Eigen functions, Generalised (Delta) functions and Green's function.

References:

1. E.A. Coddington and N. Levinson; **Theory of Ordinary Differential Equations** (Tata McGraw Hill)
2. E.L. Ince; **Ordinary Differential Equations** (Dover Publication)
3. G.F. Simmons; **Differential Equations with Applications and Historical Notes** (Tata McGraw Hill, New Delhi)
4. I. Stackgold; **Green's Function and Boundary Value Problem** (Wylie Inter Science)

MODULE 205 (GROUP: A): CLASSICAL MECHANICS- II (25 Marks)

1. Rotating frame, Coriolis force, simple examples. Principle of least action. Canonical transformation, Generating function. Poisson brackets, equations of motion. Hamilton- Jacobi equation, Hamilton's principle function, Hamilton's characteristic function, Action-angle variables, Liouville's theorem.

References:

1. H. Goldstein; **Classical Mechanics** (Addison Wesley), 1950
2. A. Sommerfeld; **Mechanics**, Academic Press, N.Y. 1952
3. E. T. Whittaker; **Analytical Dynamics** (Cambridge University Press)
4. F. Gantmacher; **Lectures on Analytical Mechanics** (MIR Publishers),1975

MODULE 205 (GROUP: B): CONTINUUM MECHANICS- I (25 Marks)

1. **Deformation and Strain :** Continuum configuration, Rigid bodies and Deformable bodies, Displacements due to deformation only, Lagrangian and Eulerian description, Displacement components and Strain components and relation between them, Infinitesimal strain, The Strain of quadric, Principal strains. Strain invariants, Cubical dilatation, Strain deviator, Different types of strain, Uniform dilatation, Simple Extension, Shearing strain, Plane strain etc., Transformation of components of strain, Compatibility Equations.
2. **Analysis of Stress :** Body forces, Surface forces and stress, State of stress in a deformable body, State of stress at a point, Transformation of stress components, Different types of stress: Normal, Shearing, Simple Extension, etc., Stress Quadric, Principal stress, Maximum and minimum shearing value, Stress deviator, Compatibility Equation, Stress boundary condition at a free surface and at surface of separation of two media. Stress equation of motion and of equilibrium, Displacement equation of motion and of equilibrium.

3. **Strain Energy Function :** Hooke's Law, Homogeneous / Transversely isotropic medium, Isotropic medium, Elastic parameters.

References

1. I.S. Sokolnikoff; **Mathematical Theory of Elasticity**, McGrawHill Book Co. Ltd.
2. T.J. Chung; **Applied Continuum Mechanics**, Cambridge University Press
3. A.E. Love; **A Treatise on Mathematical Theory of Elasticity**, McGrawHill Book Co. Ltd. 1956.
4. Y.C. Fung; **Foundations of Solid Mechanics**, Prentice Hall, 1965.
5. L.D. Landau & E.M. Lifshitz; **Theory of Elasticity** (Vol. 7 of Theoretical Physics, Pergamon Press.
6. George E. Mase; **Continuum Mechanics**, Schaum's Outline Series, McGrawHill.
7. H. Filonenko-Borodich; **Theory of Elasticity** (Translated from Russian, Peace Publishers, Moscow.
8. S.P. Timoshenko and J.N. Goodier; **Theory of Elasticity**, McGrawHill.

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MODULE 205 (GROUP: B): OPERATIONS RESEARCH - I (25 Marks)

1. **Queueing Theory (Theory of Waiting Lines):** Introduction, Queueing System, Queue disciplines FIFO, FIFS, LIFO, SIRO, FILO etc. The Poisson process, (Pure birth process), Arrival distribution theorem, Properties of Poisson process, Distribution of inter arrival times (exponential process), Markovian properties of inter arrival times, Pure death process (Distribution of departures), Derivation of service time distribution, Analogy of exponential service times with Poisson arrivals, Erlang service time distribution, Kendals notations, Probabilistic queueing models (M=M=I) : (1=FCFS), General Erlang model, (M=M=I) : (N=FCFS); (M=M=S) : (1=FCFS) and their properties.
2. **Project Scheduling and Network Analysis:** Project Scheduling by PERT and CPM, Construction of a network, Fulkerson's i/j rule, Errors and dummies in Network, Critical path analysis, Forward and backward pass methods, Floats of an activity, Project costs by CPM, Crashing of an activity, Crash-cost slope, Time estimates of for PERT, Probability of completion of a Project within scheduled time.

   References:

   1. S.D. Sharma; **Operations Research**
   2. Kanti Swarup, P.K. Gupta and Manmohan; **Operations Research**
   3. Sasieni Maurice, Arther Yaspan, Lawrence Friedman; **OR methods and Problems**
   4. H.S. Taha; **Operations Research**
   5. T.L. Satty; **Operations Research**
   6. Wagner; **Principles of Operations Research** (PH)
   7. Sasievir, Yaspan, Friedman; **Operation Research: Methods and Problems** (JW)
   8. J.K. Sharma; **Operation Research-Theory and Applications**
   9. Schaum's Outline Series - **Operation Research**
   10. Hillie & Liberman; **Introduction to Operation Research.**

MODULE 301 (GROUP- A): PARTIAL DIFFERENTIAL EQUATIONS (25 Marks)

1. Second Order Partial Differential Equations, Classification, Elliptic, Parabolic and Hyperbolic types, General solution of linear partial differential equations with constant coefficients, Transformation to Normal forms.
2. Fundamental solution of Linear differential operations, Laplace, Heat conduction and Wave operator (dimension n= 1,2,3)
3. Second order hyperbolic type equation, D' Alembert's solution.

4. Boundary value problem for elliptic type equations, Separation of variables for Laplace and Poisson Equation, Green's function technique, Dirichlet and Neumann's problem.
5. Initial Boundary value problem for heat conduction equation and Wave Equation.

References:

1. I.N. Sneddon; **Elements of Partial Differential Equation**, McGraw Hill, 1957
2. S.T. Copson; **Partial Differential Equation**, CUP, 1975
3. I. Stackgold; **Green's Function and Boundary Value Problem** (Wiley Inter Science)
4. K.S. Rao; **Introduction to Partial Differential Equation**, Prentice Hall, New Delhi, 1997.

MODULE 301 (GROUP- B): INTEGRAL TRANSFORM & INTEGRAL EQUATIONS (25 Marks)

1. Fourier transform in L_1 space, analytic properties, Riemann-Lebesgue theorem, Fourier inversion formula, Parseval's theorem, Convolution theorem, Fourier transform in L_2 space, Planchevel's theorem.
2. Laplace transform of a function of exponential order, its derivative and integral, Properties of Laplace transform, Inverse formula, Bromwich Integral, Convolution theorem
3. Application of integral transform to differential equation.
   Selected topics from:
   Integral equation, Fredholm type and Voltera type (equation of special kind). Reduction of initial value/boundary value problem to Voltera type/ Fredholm type integral equations. Homogeneous Fredholm integral equation of second kind, Eigen values and eigen funations, Adjoint kernel, Inhomogeneous Fredholm integral equation.

References

1. V.Widder Davis; **Laplace Transform**
2. V Curchil Ruel; **Operational Mathematics**
3. N.W. Mclachlan; **Complex Variable Theory and Transform Calculas with Technical Application**
4. E.C. Tichmarsh; **Theory of Fourier Integrals**
5. I. N. Sneddon; **Fourier Transform**
6. W. V. Lovitt; **Linear Integral Equation**
7. I. G.Petrovsky; **Lectures in the Theory of Integral Equation**
8. D. Parker and D. S. G. Stirling; **Integral Equations**
9. V.Vladimirov; **Equations of Mathematical Physics**
10. R.Courant and D.Hilbert; **Methods of Mathematical Physics**
11. R.Kress; **Linear Integral Equation**
12. H.Hochstadt; **Integral Equation**
13. A. S. Krasnov, A.Kisilev, G.Makarenko; **Problems and Exercises in Integral Equation**
14. Gerald B. Folland; **Fourier Analysis and its Application**

MODULE 302 (GROUP- A): DISCRETE MATHEMATICS- II (25 Marks)

1. Logic : First order Predicate Logic, Interpretation, Satisfiability and Validity of a formula models, Elementary model checking, Axiomatic approach to first order logic, Theorems, Derivations, Prenex normal form.
2. Formal language and Automata: Automata theory: Finite automata, regular languages, regular expressions, equivalence of deterministic and nondeterministic finite automata, minimisation of finite automata,

3. Kleene's theorem, pumping lemma, MyhillNerode theorem, Phrase structure grammar, Characterization of regular languages by regular grammars.
4. Relation between logic and Automata.

References

1. E. Mendelson; **Introduction to Logic**, AP
2. Margaris; **First order Mathematical Logic**, Blaisdell Publishing Co.
3. D.S. Malik and M.K. Sen; **Discrete Mathematical Structures, Theory and Applications**, Thompson.
4. J.E. Hopcroft and J.D.Ullman; **Introduction to Automata Theory, Languages and Computation**,
5. H.R.Lewis and C.H.Papadimitriou; **Elements of the Theory of Computation.**

MODULE 302 (GROUP- B): NUMERICAL ANALYSIS- I (25 Marks)

1. **Computer Number System :** Control of Round-off Errors, Instabilities-Inherent and Induced, Hazards in Approximate Computations, Well-posed and Ill-posed Problems, The Direct, Inverse and Identification Problems of Computation.
2. **Numerical Solution of Systems of Linear Equations:** Diagonal Dominance, Triangular Factorisation Methods, Matrix Inversion Method, Operation Counts, Iterative Methods – Jacobi Method, Successive-Over-Relaxation (SOR) Method, Solution of Over-Determined System of Linear Equations : Least Square Method, III Conditioned Matrix.
3. **Eigen Values and Eigen Vectors of Real Matrices :** Power Method with Shifting, Deflatation Method, Jacobi's Method for symmetric Matrix(Algorithm only).
4. **Solution of Non-Linear Equations:**
   i) **Single Equation:** Modified Newton-Raphson Method for real roots, simple or repeated, Aitken's δ^2-method and Steffensen's method.
   ii) **Roots of Real Polynomial Equations:** Sensitivity of polynomial roots, Hua's Theorem, Bairstow's method of Quadric factors, Quotient-difference method (algorithm only), Graeffe's root squaring method.
   iii) **Non-Linear System of Equations:** Newton's Method, Quasi-Newton's Method.

References

1. A Ralston , **A First course in Numerical Analysis,** Mc Graw -Hill
2. S D Conte and CDe Boor **Elementary Numerical Analysis : Algorithimic Approach** Mc Grog Hill,
3. Atkinson K E **An Introduction to Numerical Analysis**, John Wiley and Sons.
4. W F Ames **Numerical Methods for PDE**, Academic Press
5. C T H Baker and C Philips , **The numerical solutions of non linear Problems**, C P oxford.

MODULE 303 (GROUP- A): CONTROL THEORY (25 Marks)

1. Control System, State variables and Control Variables, Open and Closed Loops, Feedback, Input and Output, Transfer Functions, Laplace Transform, z-Transform
2. Controlled System and Solution, Linear Control System, Controllability, Observability, Linear Feedback.
3. Stability, Algebraic criteria for Linear System, Routh-Hurwitz Criterion, Nyquist Criterion, Lyapunov Theory, Optimal Control Problem, Performance Indices, Pontryagin's Principal, Linear Regulator.

References :

1. S. Barnett; **Introduction to Mathematical Control Theory**, Clarendon Press, Oxford, 1975.
2. R.E. Kalman, P.L.Fall and M.A. Akrib; **Topics in Mathematical System Theory**, McGrawHill, 1969.
3. O.I.Elgerd**, Control System Theory**, McGrawHill, 1967.

MODULE 303 (GROUP: B): CONTINUUM MECHANICS- II (25 Marks)

1. **Conservation of Mass :** The Continuity equation, Momentum Principles, Equation of Motion.
2. **Invicid Incompressible Fluid :** Field equations, Circulation, Propagation of small distances, Mach number and cone, Bernoulli's Equation, Irrotational motion, Velocity potential, Stream function, Stream line, Complex velocity, Bernoulli's equation, Pressure density.
3. **Viscous Incompressible Fluid Flow :** Field equation, Boundary condition, Reynold's number, Vorticity equation, Circulation, Flow through parallel plates, Flow through pipes, Shear flows.

References

1. S.I. Pai, **Viscous Fluid Theory**, Princeton.
2. F. Chorlton, **Text Book of Fluid Dynamics** CBS Publ.
3. H. Lamb, **Hydrodynamics**, Dover Publications.
4. L.M. Millne-Thomson, **Theoretical Hydrodynamics**.
5. L.D. Landau & E.M. Lifshitz, **Theory of Elasticity (Vol. 7 of Theoretical Physics, Pergamon Press**.
6. George E. Mase, **Continuum Mechanics**, Schaum's Outline Series, McGrawHill.

--OR—

MODULE 303 (GROUP: B): OPERATION RESEARCH- II (25 Marks)

1. **Replacement problem Models:**
   Replacement problem, Types of Replacement problems, Replacement of capital equipment that varies with time, Replacement policy for items where maintenance cost increases with time and money value is not considered, Money value, Present worth factor (pwf), Discount rate, Replacement policy for item whose maintenance cost increases with time and money value changes at a constant rate, Choice of best machine, Replacement of low cost items, Group replacement, Individual replacement policy, Mortality theorem, Recruitment and promotional problems.
2. **Inventory Problems:**
   Introduction, Inventory problems, Inventory parameters, Variables in inventory problems, Controlled and uncontrolled variables, Classification of inventory models, Deterministic elementary inventory models, Economic lot size formula and its properties, Problems.

References:

1. S.D. Sharma-**Operations Research**
2. Kanti Swarup, P.K. Gupta and Manmohan-**Operations Research**
3. Sasieni Maurice, Arther Yaspan, Lawrence Friedman-OR **methods and Problems**
4. H.S. Taha-**Operations Research**
5. T.L. Satty- **Operations Research**
6. Wagner-**Principles of Operations Research (PH**)
7. Sasievir, Yaspan, Friedman- **Operation Research: Methods and Problems** (JW)
8. J.K. Sharma-**Operation Research-Theory and Applications**
9. Schaum's Outline Series-**Operation Research**
10. Hillie & Liberman- **Introduction to Operation Research**

MODULE 304 (GROUP- A): ACTUARIAL SCIENCES (25 Marks)

1. Generalised Cash –Flow model of financial transactions : Zero coupon Bond, a fixed interest security, index-linked security, cash on deposit, equity, interest only loan, repayment loan, annuity certain.
2. Time value of money using the concept of Compound interest and discounting. Real and money interest rates, force of interest.
3. Present value and accumulated value of a stream of equal or unequal payments using specified rates of interest.
4. Derivation of important compound interest functions including annuities certain; equation of value; loan
5. Loss models: exponential, gamma, Generalised Pareto, normal, lognormal, Weibull, Burr distribution, moments, moment generating function, Cumulant generating function . Estimation of Parameters: MLE, Bayesian estimators under different loss functions.
6. Life table: Hazard rate , force of mortality, survival function. Mortality curve-Gompertz and Makeham.
7. Risk - Measurable risks, Determination of premium using utility function.
8. Risk Models: Short term contract, Insurable risk; Types of insurance cover –Liability, Property damage, Financial loss, Fixed benefits.
9. Reinsurance.
10. Ruin Theory.

<u>**References:**</u>

1. **Actuarial Mathematics** : N.L. Bowers, H.U. Gerber, J.C. Hickman.
2. **Modern Actuarial Theory and Practice**, P. M. Booth, R. G. Chadburn, D R Cooper, Chapman and Hall, 1999
3. **Life Contingencies**, E.T.Spurgeon, Cambridge University Press, 1972

MODULE 304 (GROUP- B): STOCHASTIC PROCESS (25 Marks)

1. A review of discreet and continuos random variables includind Binomials, Poisson, uniform, exponential, and normal variables. Conditional probability.
2. Introduction of Stochastic processes. Classification of stochastic processes according to state space and time domain.markov chain. Polyaqns urn models . homogeneous chains. Transition probabilities random walks Gambler's ruin problem. Queuing model. Order of a Markov chain. Higher translation probabilities – Chapman – Kolmogorov equation. Transition matrix of m-step transitions. Generized Bernoulli trials. Markov Bernoulli chain. Correlated Random Walk.
3. Classification of states and chains. Class property. Transient and Persistent states. First Entrance Theorem. First Passage Time Distribution Ergodic state of a Markov chain. Condition of a Persistant state.
4. Poisson process. Brownian motion and Wiener process.

<u>**References:**</u>

1. Feller W**. An introduction to probability theory and its application**, Wiely Eastern, New Delhi.
2. Hoel P. G. Port, S.C. and Stone, C.J. **Introduction to scholastic process**, University Book Stall, New Delhi

MODULE 305: ELECTIVE PAPER (50 Marks): Detailed syllabi for all the Elective Papers are given after Module 405.

MODULE 401 (GROUP- A): GRAPH THEORY (25 Marks)

1. **Graphs and Digraphs:** Graphs, Geometric Representation, Simple Graph, Degree of a vertex, Euler's hand-shaking theorem, Directed graph, In-degree, out-degree.
2. **Sub graph and Isomorphism of graphs:** Sub graph, Deletion of a vertex/edge from a graph, Isomorphism of graphs, Walks, Paths, Circuits, and Cycles.
3. **Connected graph, Bipartite graph:** Connected graph, Component of a graph, Basic properties, acyclic graph, Complete graph, Bipartite graphs, Complete bipartite graphs, Necessary and sufficient condition for a bipartite graph.
4. **Eulerian and Hamiltonian graphs:** Euler Circuit, Necessary and sufficient condition for a graph to be Eulerian, Konigsberg bridge problem, Hamiltonian graph, Dirac's theorem (Statement only), Ore's theorem (Statement only), Applications.
5. **Tree:** Tree, Forest, Basic properties, weighted graph, Minimal spanning tree, Kruskal's algorithm for a minimal spanning tree.
6. **Planar graph:** Planar graph, Face-size equation, Euler's formula, Non-planarity of the graphs K_5, $K_{3,3}$.
7. **Matrix representation of a graph:** Adjacency matrix, Incidence matrix, Properties.

Reference Books:

1. Jonathan Gross and Jay Yellen: **Graph theory and its applications**- CRC Press (USA)
2. D.S. Malik and M.K. Sen: **Discrete Mathematical Structures**- Thomson, Course Technology (USA)
3. Frank Harary: Graph theory-Addison- Wesley Pub. Company
4. Narsingh Deo- **Graph theory with applications to Engineering and Computer Science**- Prentice Hall.
5. Robin J. Wilson: **Introduction to graph theory**-Pearson Education Ltd.
6. John Clark, Derek Allan Holton: **A First look at Graph theory**

MODULE 401 (GROUP- B): NUMERICAL ANALYSIS- II (25 Marks)

1. **Polynomial Interpolation:** Weierstrass' Approximation Theorem (Statement only) Divided differences, Divergence of sequences of Interpolation Polynomials for equi-spaced Interpolation Points, Piecewise Polynomial Interpolation : Cubic Spline Interpolation, Convergence Properties (Statement Only)
2. **Approximation of Functions :** Least Square Polynomial Approximation with Orthogonal Polynomials, Chebyshev Polynomials, Lanczos Economization, Harmonic Analysis.
3. **Numerical Integration :** Problem of Approximate Quadrature, Round-off Errors and Uniform Coefficient Formulae, Gauss- Legendre and Gauss-Chebyshev Quadratures, Euler-Maclaurin Summation Formula, Richardson Extrapolation, Romberg Integration, Simpson's Adaptive Quadrature, Fredholm Integral Equation, Double Integrals, Cubature Formula of Simpson Type Improper Integrals.
4. **Selected Topic from : (Any one)**
   a) **Numerical Solution of Initial Value Problems for ODE:**
      First Order Equation: Multi-Step Predictor-Corrector Methods – Adams-BAshforth Method, Admas-Moulton Method, Milne's Method, Convergence and stability, Higher Order Equations, RKF-4 Method, Stiff Differential Equations.
   b) **Two-Point Boundary Value Problems for ODE:** Finite Difference Methods, Shooting Method.

c) **Numerical Solution of Partial Differential Equations by Finite Difference Methods:** Parabolic Equation in one Dimension (Heat Equation), Explicit Finite Difference Method, Implicit Crank-Nicolson Method, Hyperbolic Equation in One-space Dimension (Wave Equation) Finite Difference Method, Method of Characteristics (Consistency, Convergence and Stability).

References:

1. Ralston, A. **A Finite Course in Numerical Analysis**, McGrawHill, NY (1965).
2. Ralston, A. and Rabinowitch, P**., A First Course in Numerical Analysis**, McGrawHill, NY (1978).
3. Conte, S.D. and deBoor, C., **Elementary Numerical Analysis: An Algorithmic Approach**, McGrawHill, NY (1980).
4. Atkins, K.E., **An Introduction to Numerical Analysis**, John Wiley and Sons, (1989).
5. Ames, W.F. **Numerical Methods for PDEs**, Academic Press, N.Y.(1977)
6. Colatz, L. **Functional Analysis and Numerical Mathematics**, Academic Press, N.Y.(1966).
7. Baker, C.T.H. and Philips, C., **The Numerical Solution of Nonlinear Problems**, C.P.Oxford(1981).
8. Ahlberg, J.H., Nilson, E.N. and Walsh, J.L., **The Theory of Splines and Their Applications**, Academic Press, N.Y.(1967)
9. deBoor, C.A., **A Practical Guide to Splines**, Springer Verlag, N.Y. (1978).
10. Froberg, C.E., **Introduction to Numerical Analysis**, Addition-Wesley Publishing Company.
11. Zurmuhl, R., **Numerical Analysis for Engineers and Physicists**, Allied Publishers Private Limited, Calcutta.
12. Fox, L., **Numerical Solution of Ordinary and Partial Differential Equations**, Oxford (1962).
13. Prentes, P.M**., Splines and Variational Methods**, Wiley-Interscience, N.Y. (1975).
14. Blum, E.K., **Numerical Analysis and Computation Theory and Practice**, Addison-Wesley Publishing Company, Inc. London (1972).
15. Pozrikidis, C**., Numerical Computation in Science and Engineering**, Oxford University Press, Inc., N.Y. (1998)
16. Wilkinson, J.H**., Rounding Errors in Algebraic Processes**, Prentice Hall, Englewood Cliffs, N.J. 1963.
17. Hildebrand, F.B., **Introduction to Numerical Analysis** (Dover 1987).

MODULE 402: ELECTIVE PAPER (50 Marks) Detailed syllabi for all the options of Elective Papers are given after Module 405.

MODULE 403: ELECTIVE PAPER (50 Marks) Detailed syllabi for all the options of Elective Papers are given after Module 405.

MODULE 404: PRACTICAL: COMPUTER PROGRAMMING, NUMERICAL PROBLEMS & SESSIONAL (50 Marks)

1. **Computer Practical (Marks - 50) :** Fundamentals of C programming / MATLAB/ Mathematica

2. **Sessional :** Find the following in C / MATLAB/ Mathematica :

[A] Elementary Problems:

(i) Sorting : (a) Bubble sort and (b) Selection sort
(ii) Sine and cosine series
(iii) Matrix Addition and subtraction
(iv) Matrix multiplication
(v) Transpose of a matrix
(vi) Summation of convergent infinite series (Approx)

[B] Numerical Problems: (Any Ten) :

(i) Find roots of real Polynomial Equations (a) Bairstow's Method
(b) Graeffe's Root Squaring Method
(ii) Polynomial Interpolation: Cubic Spline
(iii) Solution of a Non-linear equation by Aitken's δ^2-Method
(iv) Solution of a system of linear equations by (a) Gauss-Jordan method
(b) Gauss-Seidel iteration method
(c) Triangularization method or LU method
(v) Solution of a system of Non- linear system of equations by Newton's Method
(vi) Inversion of a matrix by Gauss-Jordan Method
(vii) Eigen value and eigenvector of real symmetric matrix by Jacobi's method
(viii) Approximation of Function (a) Least Squares and (b) Chebyshev Polynomial
(ix) Numerical Integration (a) Gauss-Legendre
(b) Gauss-Chebyshev quadrature
(c) Romberg
(x) Numerical Solution of Initial Value Problem for ODE: First order Equation: Multistep
(a) Adams-Bashforth Method
(b) Adams-Moulton Method
(xi) Two point Boundary Value Problem for ODE (a) Shooting Method
(xii) Numerical Solution of Partial Differential Equation by Finite Difference Method: Parabolic Equation in one Dimension (Heat Equation)

References:

1. **Programming with C:** Byron Gottfied (Schaum's Series / Tata McGraw Hill)
2. **Let us C:** Yashvant Kanetkar (BPB Publication)
3. **Programming in ANCI** C: E. Balagurusamy
4. **C-Language and Numerical Methods**: C. Xavier (New Age International Private Limited)
5. **The *Mathematica* Book,** 5th Edition, Stephen Wolfram, 2003
6. **Students introduction to Mathematica**, B.F. Torrence and E.A. Torrence,2nd edition.
7. **An introductory programming with Mathematica**, P., Wellin, S. Kamin and R. Gaylord, 3rd edition, Cambridge University Press
8. **Dynamical Systems with applications using Mathematica**, Stephen Lynch, 2007, Birkhauser, Boston.
9. **Introduction to MATLAB with numerical preliminaries**- A. Stanoyevitch, 2005, Wiley.
10. **An Introduction to Numerical Ordinary and Partial Differential Equations Using MATLAB**, A. Stanoyevitch, Wiley, 2005.
11. **An Introduction programming and Numerical Methods in MATLAB**, S.R. Otto and J.P. Denier, 2005, Springer.
12. **MATLAB a practical introduction programming and problem solving**, S. Attaway, 2nd edition, 2012, Elsevier.

MODULE 405: PROJECT WORK AND COMPREHENSIVE VIVA (50 Marks)

ELECTIVE PAPERS

MEP1-*** - ADVANCED OPTIMIZATION AND OPERATION RESEARCH

MEP1-305A : Advanced Optimization (25 Marks)

1. Convex Sets, Convex Hulls, Closure and Interior of a Convex Set, Separation and Support of Convex Sets, Convex Cones and Polarity, Polyhedral Sets, Extreme Points and Extreme Directions, Linear Programming and the Simplex Method.
2. Convex Functions: Definitions and Basic Properties, Subgradients of Convex Functions, Differentiable Convex Functions, Minima and Maxima of Convex Functions, Generalizations of Convex Functions.

MEP1-305B : Operation Research (25 Marks)

1. **Inventory Control Theory:** Procurement and inventory operations, cost parameters – item cost, holding cost, shortage cost, setup cost. Designations of procurement and inventory systems. Model I: Optimum production quantity with constant demand, with or without lead time, no shortage and instantaneous production. Reorder points with lead time known exactly; effect of setup cost being item dependent; round-off rule minimizing the penalty from the requirement that the optimum production quantity be a nonnegative integer. Model II: Economic lot size with different rates of demand in different cycles. Model III: Economic lot size with finite rate of replenishment and no shortage. Model IV: Economic lot size with fixed scheduling period and finite shortage cost. Model V: Economic lot size with variable scheduling period and finite shortage cost. Model VI: Economic lot size with finite rate of production and finite rate of replenishment.

   Probabilistic Models: Model I(a): Discrete version of instantaneous demand, no setup cost model. The Newspaper Boy Problem. Model I(b): Continuous version of instantaneous demand, no setup cost model. The Baking Company Problem.

2. **Theory of Replacement :** Types of Replacement Problems. Replacement of deteriorating items – costs to be considered, best time of replacement. Replacement policy of items whose maintenance cost increases with time, but money value is not counted when time is a discrete / continuous variable. Applications. Money value over time Replacement policy for items whose maintenance cost increases with time and the money value changes at a constant rate: (i) To find the present worth of total expenditure, (ii) Method I: When the maintenance cost increases with time and the money value decreases at a constant rate. (ii) Method II: The amount to be spent is borrowed at a given rate of interest under the condition of paying it back in pre-decided number of instalments. How to select the best one of two machines? Applications.

   Group Replacement Policy: Theorem: (a) One should group replace at the end of t-th period if the cost of individual replacement for the t - th period is greater than the average cost per period through the end of t periods. (b) One should not group replace at the end of t – th period if the cost of individual replacements at the end of $(t-1)$-th period is less than the average cost per period through the end of t periods. Application on mortality rate of light bulbs. Application on a system in which all items are new initially. Each item has a probability p of failing before the end of first month of life and a probability *q = 1-p* of failing immediately before the second month of life. If all items are replaced as they fail, to find the expected number of failures *f(x)* at the end of month *x*. To discuss the cases: (a) A group replacement policy at the end of each month is most profitable. (b) A group replacement policy at the end of every other month is most profitable. (c) No group replacement policy is better than a policy of individual replacement.

MEP1-402 : Advanced Optimization (50 Marks)

1. The Fritz-John and the Kuhn-Tucker Optimality Conditions : Unconstrained Problems, Problems with Inequality Constraints, Problems with Inequality and Equality Constraints.
2. Constraint Qualifications: The cone of tangents, Other Constraint Qualifications, Problems with Inequality and Equality Constraints.
3. Lagrangian Duality and Saddle Point Optimality Conditions: The Lagrangian Dual Problem, Duality Theorems and Saddle Point Optimality Conditions, Properties of the Dual Function, Solving the Dual Problem, Duality Gap, Linear and Quadratic Programs.

MEP1-403 : Operation Research (50 Marks)

1. **Queueing Theory :** Introduction. Queueing System: (a) The input or arrival pattern, (b) The output or service pattern. (c) The queueing discipline. Balking, Reneging, Priorities, Jockeying. Queueing Problem: (a) Probability distribution of queue length. (b) Probability distribution of waiting time of customers. (c) The busy period distribution. Transient and Steady States. Kendall's and Lee's notations for representing queueing models. Probabilistic Queueing Models: (A) Model I (Erlang Model): (M/M/1): (α / FIFO) (B) Model II (M / M/ 1): (α / FIFO): General single station queueing model. (C) Model III: (M/M/1): (N/FIFO). (D) Model IV: (M/M/K): (α / FIFO). Traffic Intensity. Poisson arrivals (pure birth process). Expected number of arrivals E{n} and Variance Var{n}. Distribution of inter-arrival times in Poisson process. Distribution of departures (pure death process). Exponential service times. Solution of Queueing Models: Model I: (M/M/1): (α /FIFO). Measures of Model I (i) Expected line length L_s ((ii) Expected queue length L_q (iii) Expected waiting time in queue W_q (iv) Expected waiting time W_s in the system (v) Expected waiting time $E\{W / W \rangle 0\}$ of a unit who has to wait. (vi) Expected length $E\{L / L \rangle 0\}$ of non-empty lines. (vii) Variance of queue length Var{n}. (viii) Probability of arrivals during the service time of any given customer. Interrelationship between L_s, L_q, W_s, W_q. Applications. Model II: (M/M/1): (α /FIFO) – General Erlang Queueing Model. Model III: (M/M/1): (N/FIFO). Model IV: (M/M/K): (α /FIFO). Measures of Model IV: L_s, L_q, W_s, W_q . Probability distribution of busy periods. Applications.

2. **Information Theory :** Introduction, Measure of Information; Units of Information – bits, nats, Hartelys; Entropy – The Expected Information; Entropy as a measure of uncertainty and its associated theorem; The Zero-Memory Information Source and its extension; Theorem: $H(S^n) = nH(S)$.

   Encoding: Definition of a Code, Block Code, Binary Code, Non-singular Code, Code Words, Uniquely Decoded Code, Instantaneous Code, Prefix; Theorem on Codes and Code Words; Construction of an Instantaneous Code; Theorem on Kraft Inequality; Average Length of a Code; Compact Code; Theorem: $L \geq \frac{H(s)}{\log r}$.

3. **Network Analysis :** Network scheduling by CPM / PERT techniques: Basic steps – planning, scheduling, allocation of resources, controlling. Representation of a network diagram – activity, dummy activity, node, event, connector, sequencing. Rules for constructing the network diagram. The critical path and slack paths of a network. Determination of critical path: earliest times and latest times, non-critical paths or slack paths, slack times, slack report, slack summary report. The CPM method – normal time and crash time schedules of a network, normal cost, crash cost, incremental cost. Applications. The Least-cost Crash-time Schedule of a network. Applications. The PERT Model – most optimistic time, most likely time, most pessimistic time, Beta Distribution of an activity, mean and variance. Normal distribution of a complete path of a network, mean, standard deviation, variance. Applications.

References :

1. **Nonlinear Programming : Theory and Algorithms**. Mokhtar S. Bazaraa, C. M. Shetty. John Willey & Sons.
2. **Optimization by Vector Space Methods**. D. G. Luenberger. John Willey and Sons, Inc.
3. **Nonlinear Programming**. O. L. Mangasarian. McGraw-Hill.
4. **Nonlinear Programming: Analysis and Methods**. M. Avriel. Dover.

MEP2 - ADVANCED REAL ANALYSIS AND ADVANCED COMPLEX ANALYSIS**

MEP2-305 : Advanced Real Analysis (50 Marks)

1. **Ordinal Numbers:** Well-ordered sets, order types, ordinal numbers, limit ordinals, Transfinite Induction, comparability of ordinal numbers, Any set of ordinal numbers is a well- ordered set, Every ordinal α can be uniquely written as α=β+n where β is a limit ordinal and n finite, first uncountable ordinal.
2. **Descriptive properties of sets:** Perfect sets, Description of a closed set in terms of a perfect set, sets of first category and second category, Residual set, characterization of a residual set in a complete metric space, points of condensation of a set, concept of Borel sets of class α,α being ordinal number and its basic properties, Density point of sets, Lebesgue Density Theorem.
3. **Functions of some special classes**: Lower and upper semi continuous functions and their properties, Baire functions of class α, $\alpha < \Omega$, Ω being first uncountable ordinal and its simple properties, comparison of Baire function of class α and Borel sets, set of points of discontinuity of Baire function of class one, approximately continuous function and some basic properties, measurability of approximately continuous function.
4. **Derivatives:** The Vitali Covering Theorem, absolutely continuous function and its simple properties, Banach –Zarecki theorem, mapping by absolutely continuous function, Dini's derivates and their simple properties, derivative of an increasing function and absolutely continuous function, integrability of approximately continuous functions, Lebesgue point of a summable function and its basic properties.

MEP2-402 : Advanced Complex Analysis (50 Marks)

1. Analytic functions and Families of analytic functions : -Hadamard's three-circles theorem, The function A(r), Borel-Caratheodory theorem, Mean values of $|f(z)|$, uniform boundedness of a sequence of functions, Equi continuity, Normal family, Hurwitz's theorem, Vitali's theorem , Montel's theorem
2. Infinite Product of Complex numbers and Complex functions:- Infinite Product, convergence, absolute convergence, Uniform Convergence M-lest for absolute and uniform convergence. Harmonic Functions :- Maximum and Minimum principle, Gauss' Mean value theorem, Poisson's

integral formula, Poisson Kernel , Dirichlet Problem for a disk, Schwarz integral formula, Harnack's inequality

3. Distribution of Zeros of analytic functions :- Jensen's formula, Poisson-Jensen formula, Jensen's inequality, the function n(r), Jensen's inequality in terms of n(r).
4. Entire Functions:- Growth properties of an entire function, order, type, convergence exponent of Zeros, Factorization of entire functions, Weierstrass' factorization theorem, Canonical Product, Borel's theorem, Hadamard's factorization theorem, Picard's little theorem, Order and type in terms of Taylor Coceffiients (Statements only), Application of the results.

MEP2-403A : Advanced Real Analysis (25 Marks)

1. **Hens tock Integral:** Gauge function $\delta: [a, b] \to (0, \infty)$, δ- fine partition of $[a, b]$, existence of δ-fine partition of $[a, b]$, definition of Hens tock Integral of a function on $[a, b]$and examples, some elementary properties of this integral, Cauchy criterion, Saks-Hens tock lemma and its applications, continuity of indefinite integral and fundamental theorem of this integral, absolute Hens tock Integral, Hens tock Integral includes Newton, Riemann and Lebesgue integrals, Absolute Hens tock Integral is Lebesgue integral, monotone and dominated convergence theorems of Hens tock Integral.
2. **General measure and integration**: set functions, measure, some properties of measure, outer measure, extension of measure, completion of measure, measure space, complete measure space, measurable functions and its basic properties, sequence of measurable functions, limit of measurable functions, almost everywhere convergence, convergence in measure, almost uniform convergence, Egoroff's theorem, simple functions, Integrable simple functions, non-negative integrable functions, integrable functions, Fatou's lemma, Monotone convergence theorem, Dominated convergence theorem, absolutely continuous measure, signed measure, Radon-Nikodym theorem.

MEP2-403B : Advanced Complex Analysis (25 Marks)

1. **Meromorphic Functions:** Value distribution and growth properties of meromorphic functions. Order, type, class of meromorphic functions. Two Main Theorems and their consequences.
2. Mittag-Leffler Theorem.

References

For Advanced Real Analysis

1. A.M.Bruckner, J.B. Bruckner & B.S.Thomson, **Real Analysis**, Prentice-Hall, N.Y, 1997
2. C.Goffman, **Real Functions**, Rinehart Company, N.Y, 1953
3. I.P.Natanson, **Theory of Functions of a Real Variable**, Vol. I, II, Fredrick-Unger Publishing, N.Y,1954.

For Advanced Complex Analysis

1. A.I. Markusherich, **Theory of functions of a complex variable,** Vol –I & Vol- II – (Prentice Hall).
2. J.B.Conway, **Functions of one complex variable.** (Narosa Pub, House).
3. A.S.B Holland, **Theory of entire function.** (Academic Press)
4. E.C. Titchmarsh, **Theory of entire function** (Oxford University Press).
5. R.P. Boas, **Entire function**.(Academic Press).
6. L.V.Ahlfors, **Complex Analysis.** (Mcgraw-Hill)
7. S.Ponnusamy, **Foundations of Complex Analysis (2nd Edition)**.(Narosa Pub House).
8. W. K. Hayman, **Meromorphic Functions**, The Clarendon Press, Oxford(1964).
9. C. C. Yang and H. X. Yi, **Uniqueness Theory of Meromorphic Functions**, Mathematics and its Applications, 557. Kluwer Academic Publishers Group, Dordrecht, 2003.

10. J. Zheng, **Value Distribution of Meromorphic Functions,** Springer-Verlag, Berlin Heidelberg, 2010.
11. R. Gorenflo and A. A. Kilbas**, Mittag-Leffler Functions, Related Topics and Applications,** Springer Monographs in Mathematics, 2014.

MEP3 - DYNAMICAL SYSTEMS**

<u>MEP3-305A</u> : Dynamical Systems-I (25 Marks)

Existence and uniqueness theorems for Cauchy initial value problems. Local and Global Solutions. Lipschitz conditions for 1D and 2 D Differential Equations.
Gronwall's inequality. Continuity with respect to initial conditions .Hyperbolic flow, Contraction, Generic Property
Canonical form of Linear Ordinary differential Equations. Autonomous and Non-autonomous Systems. Solutions of simultaneous ordinary linear equations of the form
dX/dt =AX +B in Matrix Formalism. Fundamental Matrix .Floquet's theorem for second order linear differential Equations with periodic coffecients.
Application of Floquet's theorem to Mathieu equations.
Fixed points stability, Lyapunov function. Lyapunov theorem
Hartman –Grobman theorem, Stable ,Unstable ,Centre Manifolds. Structural Stability.

<u>MEP3-305B</u> : Dynamical Systems-II (25 Marks)

1. Discreet time dynamical system. Maps, orbits, fixed points, periodic points and their stabilities. Atractors and repellers.
2. Graphical representation of an orbit. Cobweb plot. Tent map, Logistic map and Skinny baker map. Basin of a sink. Sensitive dependence on initial conditions.
3. Itineraries. Transition graphs. Arrangement of subintervals of itinerary.
4. Two – dimensional maps. He'non map. Saddle fixed points. Linear maps. Many dimensional maps. Nonlinear maps. Hyperbolic fixed points. Conditions for stable, unstable and saddle points.
5. Inverse maps. Stable and Unstable manifolds. Flip saddle. Homoclinic point.
6. Lyapunov number and lyapunav exponent of the orbit. Asymptotically periodic orbits. Chaotic orbits of one dimensional map. Binary expansion of anumber and chaotic orbits of simple maps. Tent map symbotic dynamics.
7. Conjugacy between the maps. Computation of Lyapunav numbers by utilizing the conjugacy between the maps. Fixed point thorem for one – dimensional map and it's consequences.

<u>MEP3-402</u>: Dynamical Systems-I (50 Marks)

Planar Dynamics. Hopf Bifurcations. Stability of fixed points in 2D and 3D Dynamical SystemsTwo dimensional Fixed Point Theorem. Periodic orbits, Limit Cycles

Poincare - Bendixon Theorem. Poincare Map, Poincare section. Invariant
Set . α and ω limit sets , wandering, and non-wandering points.
Basin of attraction. .Lotka-Volterra competition model.
Non-linear Oscillators, Relaxation Oscillators, Forced Oscillators, van der Pol
Oscillator, Duffing Oscillator ,Pendulum Resonance.
Multiple Time Scale and Averaging Method ,Derivative Method
Lorentz Equations and Rossler Equations.

Solitary Waves and Solitons: Basic concepts .Standard non-Linear Partial Differential Equations ,Methods of solutions.
Solitary wave solutions of KdV ,sine-Gordon, Burger and Non_Linear Schrodinger Equations. Hopf Cole-transformation, Backlund transformation ,Miura transformation .Liouville equation, Heat Equation.

MEP3-403 : Dynamical Systems-II (50 Marks)

1. Basins of attraction. Theorem for findings basins for sinks of one dimentional maps. basins for sinks of two dimentional maps Swararzian deribative and basins.
2. Fractals. Cantor Set. Base- 3 Representation of a number and middle third cantor set. Probabilistic construction of fractals. .affine constraction maps and iterated function systemin many dimentional euclidian space.Skinny baker map. Sierpinski gasket and karpet.Fractals from deterministic system.slope – 3 tent map and middle third cantor set. Self similarty of the Henon attactor. Fractal basin boundaries. Mandel boart set. Julia set. Fractal dimention.Box counting dimention and liapunav dimention. Simplyfication for box counting dimention. Computations of box counting dimention of map. Relation between box counting dimention and measure of asset.
3. Invariant measure. Invariant measure of W map. Orbital density. Orbital densities of a simple map. Erogative Orbits. Variational equation for a dynamical System.
4. Chaos in two or more dimentional maps.. Lyapunov exponent for orbit of many dimentional map. Chaotic orbit.Skyny baker map and cat map and other two dimentional map and their chaotic orbit.Two dimentional fixed point theorem. Bifurcation and periodic doubling bifurcation and saddle note. Bifurcations diagram . Transcritical bifurcation.Pitch fork bifurcation .
5. Stable manifold theorem .Hetero clinic point. Sarkowski theorem.

References:

1. K. T. Alligood,T.D.Sauer,J.A.Yorke, CHAOS**: An introduction to Dynamical Systems**.E. R. Schneirman
2. **An Invitation to Dynamical Systems** (Dover , Freely available on internet.)
3. F.J.Murray and K. S. Miller Existence **Theorems for ordinary differential equations** (Dover publication N.Y)
4. V.I. Arnold ,**Mathematical Methods of Classical Mechanics** :Springer , N.Y (1978)
5. M.W.Hirch,S.Smale,R.L Devaney ,**Differential Equations**
6. **Dynamical Systems and An Introduction to Chaos**: Elsvier Academic Press ,2nd Edition (2004)
7. V.Arnold,**Ordinary Differential Equations** ,M.I.T Press,Cambridge (1873)
8. S.H.Strogatz,**Non-Linear Dynamics and Chaos**: Addison –Wesley Publication Company (1994)
9. R.C.Hilborn**,Chaos and Non-Linear dynamical systems**
10. R. l. Devancy - **An introduction to Chaotic Dynamical System,** Wesley ,1989
11. 2. D. K. Arrowsmith and F. M Place**- An introduction to , Dynamical System**, Cambridge university press, 1990
12. V. I. Arnold—**Dynamical system V – Bifurcation theory and catastrophy theory**, springer-Verlag, 1992
13. K.T.Alligood, T.D Souar and J.a Yorke—Chaos **, An introduction to Dynamical System**. Springer, 1997
14. E.N. Lorenj—**The essence of Chaos,** The university of Washinton Press,1993
15. E. Ott---**Chaos in Dynamical system**, Cambridge University Press.1993

MEP4 - LOGIC, AUTOMATA AND ADVANCED ALGEBRA**

MEP4-305 : Logic (50 MARKS)

1. Predicate calculus, completeness theorems of predicate calculus; Godel numbers, recursive functions, Representability theorem. Godel's First Incompleteness Theorem.
2. Modal Logics**,** K, T, S_4, B, S_5. Syntax and Semantics (Kripke and Algebraic); Soundness, Completeness and Decidability.
3. Many-Valued logics, Formal Systems, Soundness and Completeness results. Elements of Fuzzy Set theory; Fuzzy Logics, formal Syntax and Semantics. Approximate Reasoning.

MEP4-402: Automata (50 MARKS)

1. Context-free grammar and languages, Chomsky normal form, pushdown automata, Context-sensitive languages, Chomsky hierarchy, closure properties.
2. Recursive, Primitive Recusive and partial recursive functions. Recursive and semirecursive (r.e.) sets, various equivalent models of Turing machines, ChurchTuring thesis, Universal Turing machines and Halting Problem. Reducibility.

MEP4-403 : Advanced Algebra (50 MARKS)

1. Finite groups, simple groups, solvable groups, simplicity of A_n.
2. *Field Theory*: Algebraic Extensions, Finite and algebraic extensions, algebraic closure,separable and inseparable extensions, Roots of polynomials, splitting field of a polynomial, finite fields.
3. *Galois theory*: Galois extensions and Galois group, fundamental theorem; Explicit examples and concrete applications of Galois theory; Roots of unity, cyclotomic polynomials and extensions, solvability by radicals.

References:

For Logic:

1. E. Mendelson – **Introduction to Logic, AP**
2. Margaris – **First order Mathematical Logic**, Blaisdell Publishing Co.
3. S. M. Srivastava – **A Course in Mathematical Logic**, Springer
4. H. Enderton **– Introduction to Mathematical Logic, AP**
5. J. Schoenfield – **Introduction to Logic, AP.**

For Automata:

1. J.E. Hopcroft and J.D.Ullman **:Introduction to automata theory, languages and computation**,
2. H.R.Lewis and C.H.Papadimitriou :**Elements of the theory of computation.**

For Advanced Algebra:

1. D.S. Dummit and R.M. Foote: **Abstract Algebra**, Wiley Ch.13, 14, 15.1 – 15.3
2. N.S.Gopalakrishnan**: UniversityAlgebra**, Wiley Eastern – Ch. 4,TIFR pamphlet on Galois theory.
3. S. Lang – **Algebra**, Addison Wesley, (Ch. 5, 6.1 – 6.7, 7.1, 8.1, 9.1);
4. I. N. Herstein **– Topics in Algebra**, John Wiley, Ch. 5.
5. N. Jacobson Basic **Algebra** 1, HBA, Ch. 4
6. G. Rotman **– Galois Theory,** Springer

MEP5** - MATHEMATICAL BIOLOGY & ECOLOGY

MEP5-305 : Mathematical Biology & Ecology (50 Marks)

1. **Dynamics of single species growth:** Introduction. Basic definitions and fundamental concepts. Exponential growth model of Malthus: Formulation, solution and interpretation, limitations. Compensation and depensation. Verhulst's Logistic Growth Model: Formulation, solution and interpretation, limitations.
2. **Application of Lyapunov theory of stability:** Application of Lyapunov theory of stability to logistic equation, general single species model, theorem on global stability, construction of Lyapunov function, Gompertz model, Schoener model, Volterra model of schooling fish species, Allee effects and Allee curves.
3. **Harvesting of single species population:**Elementary dynamics of exploited populations. Harvesting of populations with logistic growth at a constant rate, maximum sustainable yield, biological overexploitation. Fishing effort, Catch-per-unit effort (CPUE) hypothesis. Generalized logistic growth models: compensation, depensation, critical depensation, minimum viable population level. Yield-effort curves: compensation, depensation, critical depensation.

MEP5-402: Mathematical Biology & Ecology (50 Marks)

1. **Dynamics of two species growth:** Dynamical system in the plane: Unstable node, stable node, saddle point, unstable focus, stable focus, center. Types of interaction between two species: neutralism, mutual inhibition competition, resource based competition, amensalism, predation or parasitism, commensalism, protocooperation, mutualism, symbiosis. General two-species Lotka-Vilterra model, local stability of the system, global stability of the system. Volterra's prey-predator model: Formulation, solution and interpretation, perturbation technique, limitations. Competing species: Formulation, solution and interpretation, perturbation technique, limitations.
2. **Bioeconomic models of open-access fishery:** The open-access fishery, Gordon's static model, opportunity cost, externality, economic overfishing, overfishing catastrophes, production function, Cobb—Douglas production function, discounting, the Schaefer model, optimal harvest policy.
3. **Bioeconomic exploitation of cohorts:** Beverton-Holt fishery model: cohort and its recruitment, natural mortality rate, fishing mortality rate, average weight of a fish, knife-edge selectivity, mesh-size parameter, sustainable yield, eumetric yield. Dynamic optimization of Beverton-Holt fishery model.

MEP5-403: Mathematical Biology & Ecology (50 Marks)

1. **Bioeconomic exploitation of cohorts:** Beverton-Holt fishery model: cohort and its recruitment, natural mortality rate, fishing mortality rate, average weight of a fish, knife-edge selectivity, mesh-size parameter, sustainable yield, eumetric yield. Dynamic optimization of Beverton-Holt fishery model.
2. **Bioeconomic explotation of forests:** Growth and aging: Introduction. Faustmann model; commercial value of a tree, optimal age of felling a tree, forest rotation problem, logging and replanting costs, site-value, sustainable economic rent, response to changes in demand, short-term and long-term effects of forest thinning. Killkki-Vaisanen model of optimal forest thinning: thinning from below, thinning from above, bang-bang and impulse controls, singular path, clear cutting the forest. Matrix model for the management of height structured forest: Height-value of trees, non-harvest vector, growth matrix, harvest vector, sustainable harvesting policy of a height structured forest, optimal sustainable yield.

3. **Bioeconomic exploitation of multspecies fishes:** Multispecies models in fishery management: Combined harvesting of two ecologically independent fish species following logistic growth, differential productivity, bionomic equilibrium, bio-technical productivity (BTP), optimal harvest policy. Combined harvesting of two competing fish species following logistic growth, selective harvesting. Gause's model of interspecific competition between two species.
4. **Mathematical models in epidemics:** Deterministic models in epidemics: Simple epidemics, general epidemics, Kermack- Mckendrick threshold theorem, recurrent epidemics, seasonal variation in infection rate, allowance for incubation period, models with undamped waves.
   Stochastic epidemic models, Yule-Furry birth process, expectation and variance of infectives, calculation of expectation using moment generating functions, stochastic epidemic models.

References:

For Ecology :

1. J.D. Murray (2001), **Mathematical Biology**, Springer, Berlin.
2. K.T. Alligood T.D. Sauer and J.A. Yorke (1999), Chaos: **An Introduction to Dynamical Systems**, Springer, Berlin.
3. P.O.Drazin (1993), **Non-Linear Systems,** Cambridge University Press.
4. P.S. Addison (2005), **Fractals and Chaos**, Overseas Press, New Delhi.(Indian Edition).
5. H.I.Freedman (1989), **Deterministic Mathematical Models in Population Ecology**, Marcel Dekkar.
6. F. Bauer and C. Castilo-Chavag (2001), **Mathematical Models in Population Biology**, Springer-Verlag.

For Mathematical Biology :

1. C.W. Clark, Mathematical Bioeconomics **: The Optimal Management of Renewable Resources**, John Wiley & Sons. NY, 1976.
2. C.W. Clark**, Bioeconomics Modelling and Fisheries Management**, John Wiley & Sons. NY, 1985.
3. N.T.J Bailey, **The Mathematical Theory of Infectious Diseasesand Its Applications**, Griffin, London, 1975.
4. N.T.J Bailey, **The Mathematical Theory of Epidemics**, Griffin, London, 1957.
5. J.D. Murray**, Mathematical Biology**, Springer, 1990.
6. C.W. Gardiner, **Handbook of Stochastic Methods for Physics, Chemistry** and Natural Sciences, Springer, Berlin, 1990.
7. T.C.Gard, Stochastic Differential Equation with Application in Population Biology, Marcel-Dekkar.

MEP6-*** - ADVANCED ALGEBRA AND ADVANCED RIEMANNIAN MANIFOLD.

MEP6-305A : Advanced Algebra(25 Marks)

1. **Module Theory:** Modules, submodules, quotient modules, homomorphisms, isomorphisms, exact sequences, free modules. Projective and Injective modules. Simple modules and semi simple modules.
   Tensor Product of modules. Modules over Principle ideal domain.

<u>MEP6-305B</u> : Riemannian Manifold(25 Marks)

1. General introduction to differential manifolds. Tangent space of a manifold at a point. Differentiable maps between manifolds. Vector Field. f- related vector field. One-parameter group of transformations on a manifold. Cotangent Space. Differential Form. Pull-back differential form. Exterior Derivative.

<u>MEP6-402</u> : Advanced Algebra(50 Marks)

1. **<u>Galois Theory</u>**.
2. **<u>Ring:</u>** Simple and primitive rings. Semisimple rings. Wedderburn- Artin Theorem. Jacobson Radical. Jacobson semisimple rings. Relation between Semisimple rings and Jacobson Semisimple rings. Group rings. Regular rings. Regularity of the matrix ring $M_2(R)$ over the field R of Real numbers. Relation between Semisimple rings and regular rings. Prime radical. Prime and semiprime rings, Matrix ring $M_n(R)$ over a prime ring R.

<u>MEP6-403</u> : Riemannian Manifolds (50 Marks)

1. Affine Connection. Torsion tensor field. Curvature tensor field. Covariant derivative of vector Field. Covariant derivative of Form.
2. Riemannian manifolds. Fundamental Theorem of Riemannian manifold. Riemann Curvature. Sectional curvature, Ricci Curvature. Parallelism of Vector Field. Goldberg Result. Weyl Conformal Curvature Tensor. Geodesic.

<u>References</u>

<u>For Riemannian Manifolds:</u>

1. W.M. Boothby; **An Introduction to Differential Manifolds and Reimannian Geometry.**
4. Kobayashi and Nomizu; **Foundations of Differential Manifolds, Vol. I & II.**
5. S. Helgason, **Differential Geometry, Lie Groups and Symmetric Spaces.**

<u>For Advanced Algebra:</u>

1. D.S. Dummit and R.M. Foote: **Abstract Algebra**, Wiley.
2. N.S.Gopalakrishnan; **UniversityAlgebra**, Wiley Eastern.
3. S. Lang; **Algebra**, Addison Wesley.
3. I. N. Herstein; **Topics in Algebra**, John Wiley.
4. N. Jacobson; **Basic Algebra** 1, HBA.
5. G. Rotman; **Galois Theory**, Springer

MEP7 - ADVANCED FUNCTIONAL ANALYSIS AND ADVANCED TOPOLOGY**

<u>MEP7-302A</u>: Advanced Functional Analysis (25 marks)

1. Convex hull of a set in a linear space, its representation theorem; Symmetric sets, balanced sets and absorbing sets in a linear space-their relations. Topological vector spaces(TVS), Translation and multiplication operators as self-homomorphisms over TVS, Local base, Basic properties of TVS. Bounded sets, compact sets in TVS. Linear operators and linear functional over TVS. Locally compact TVS and its finite dimensionality. Minkowski functionals, their continuity in TVS, Semi norms, locally convex TVS; Kolmogorov theoerem on normability of a TVS.

2. Banach-Stein Havs theorem in Normed linear space(NLS) and its application. Dual space X^*, $X^{**}$of a NLS X, Separability of X^*, Riesz representation theorem for a bounded linear functional over spaces R^n and sequence-space l_p (1<p<∞).

MEP7-302B : Advanced Topology- Algebraic Topology (25 Marks)

1. Homomorphisms in topological space X, paths, equivalent paths in X; Homotopy, Homotopy classes, Homotopic mappings and properties; contractible spaces, retractions, fundamental group, Isomorphism of fundamental groups, homotopy groups, invariance. Covering spaces, local homomorphism, covering maps and properties. Fundamental group of covering space. Simplexes, complexes, triangulations, simplicial mapping, topological dimension.
2. Brower fixed point theorem, finitely generated abelian groups, chains, boundaries, cycles, homology groups. Connected complexes, singular homology, boundary operator.

MEP7-402 : Advanced Functional Analysis (50 Marks)

1. Riesz representation theorem for a bounded linear functional over Banach space C[0,1] with sup norm and over a Hilbert space. Closed convex set in Hilbert space having element of minimum norm; sesquilinear form in H and its representation.
2. Weak convergence and strong convergence in NLS X, their coincidence in finite dimensional NLS. Best approximation in NLS, existence of best approximation criterion, strictly convex norm; uniqueness of best approximation ; Resolvant $\rho(T)$ and spectrum $\sigma(T)$ of a bounded linear operator T over NLS X; Expansion of $(I-T)^{-1}$; Properties of $\sigma(T)$.
3. Hilbert adjoint and self-adjoint operators on Hilbert space H; Projection operators, unitary and normal operators over H, their properties; Positive operators and square root lemma. Banach algebra X with identity, Invertible elements in X. Group structure of invertible elements; Topological div of zero in X. Analytic property of resolvent function, spectral radius of an element in X- its formula. Spectrum $\sigma(x) \neq \emptyset$. Gelfand-Mazul theorem in X; Ideals and maximal ideal in X; Gelfand theory on maximal Ideal space.
4. Weak $*$ topology in NLS X, its Hausdorff property, Weak $*$ compactness and Banach alagulii theorem of Weak $*$ compactness of closed unit ball in X^*.

MEP7-403 : Advanced Topology-Topological Group (50 Marks)

1. Topological group; Neighbourhoods of the identity, closure of a set and its representation, separation axioms – separation theorem and consequences. Subgroup of a topological group; Normal subgroup; Locally Euclidean group, Homomorphisms between topological group and Lie-group. Locally compact topological group, compact topological group, dual groups and charactes groups with their representations.
2. Continuity of inverse operation in a Banach algebra, Algebraic and topological properties of a set of invertible elements; Spectral radius formula; Ideals in algebra, space of maximal ideals, compactness property; Fourier transformation of functions in L_1 and L_2; their algebra. Relation between transforms and conjugates.

References

For Advanced Functional Analysis:

1. W Rudin; **Functional Analysis**
2. Schaefes; **Topological Vector Spaces**
3. Kryszig; **Functional Analysis**
4. Bachman and Narici; **Functional Analysis**
5. Brown and Page; **Functional Analysis**
6. Bachman; **Abstract Harmonic Analysis**

For Advanced Topology:

1. A. H. Wallace; **Algebraic Topology**
2. F. H. Spanier; **Algebraic Topology**
3. W. Fulton; **Algebraic Topology**
4. W. J. Thron; **Topological Structures**
5. Dugundji; **Topology**
6. L. Pontrjagin; **Topological Groups**
7. T. Hussain; **Introduction to Topological Groups**

MEP8 - COMPUTATIONAL FLUID DYNAMICS**

MEP8-305 : Basics of Fluid Dynamics –I (50 marks)

1. Concepts of fluid flows: fluid flow, uniform flow and steady flow, frame of reference, real and ideal fluids, compressible and incompressible fluid one, two and three dimensional flow, analyzing fluid flow, motion of fluid particle, laminar and turbulent flow.
2. Kinematics : material and special descriptions, path line, streamlines, streaklines, differentiation w.r.t. time, state of motion, rate of change of line, surface and volume elements, rate of change of material integrals.
3. Fundamental Laws of Continuum Mechanics: conservation of mass – Equation of continuity, balance of momentum, balance of angular momentum, balance of energy, balance of entropy, thermodynamics equations of state.
4. Constitutive relation for Fluids:
5. Equations of Motions for particular Fluids:

a) Newtonian Fluids—The Navier- Stokes Equations, Vorticity Equations, Effects of Renold's number.
b) Inviscid Fluids—Eular's Equation, Bernoulli's Equation, initial and boundary conditions, simplification of the Equations of motion.

6. Motion in Two Dimension (Source, Sinks, Doublets):Lagrangian Stream function, Irrotational motion in two dimension, complex potential,Definition of source, complex potential due to a doublet, Image w.r.t. a straight line, image w.r.t. a circle, Circle Theorem of Milne-Thomson, Blasius Theorem and related problems.

MEP7-402: Computational Fluid Dynamics-II (50 Marks)

1. Motion in two Dimension (Motion of a Circular Cylinder): Boundary condition for stream function, general motion of a cylinder, Motion of a circular cylinder, application of circle theorem, initial motion between two co-axial cylinders, kinetic energy of liquid, steaming and circulation about a fixed circular cylinder, Kelvin's circulation theorem, equation of motion of a circular cylinder with circulation; and related problems.
2. Stoke's Steam function: Stoke's function, properties of Stoke's function, Irrotational motion, solution for ψ, motion of a solid of revolution along a axis, φ and ψ due to three dimensional source, ψ due to line source, Image of a source relative to a sphere, image of doublet relative to sphere and related problems.
3. General Theory of Irrotational Motion: Irrotational Motion, K.E. of finite liquid, K.E. of infinite liquid, Kelvin's minimum energy theorem, Mean value of φ, Green's theorem; and related problems.
4. Vortex Motion: Vortex filaments, complex potential due to a vortex of strength $+k$, motion due to m vortices
5. Single vortex in the field of several vortices, two vortex filaments, image of a vortex w.r.t. a plane, Image of a vortex w.r.t. a cylinder; and related problems.

6. Laminar Unidirectional Flow: Steady flows--- Couette Flow, Couette-Poiseuille flow, flow down a inclined plane, flow between two rotating concentric cylinders, Hagen-Poiseuille flow.
7. Boundary Layer Flow: Drag and lift, Prandtl's boundary layer theory, boundary layer equations, Von Karman's integral equation, Von Karman's momentum integral equation, flow parallel to a semi-infinite plate, Von Karman- Pohlhausen method, Von Karman's momentum theorem and boundary layer.

<u>MEP7-403</u>: Computational Fluid Dynamics-III (50 Marks)

1. Computational Fluid Dynamics: Introduction, Finite Difference Method, Finite Element Method, Finite Volume Method, Problems on Incompressible Viscous Fluid Flow.
2. Turbulence
   a) Introduction , b) Averages, c) Correlations and Spectra, d) Averaged Equations of Motion-- Mean Continuity Equation, Mean Momentum Equation, Reynolds Stress, Mean Heat Equation, e) Kinetic Energy Budget of Mean Flow, f) Kinetic Energy Budget of Turbulent Flow, g) Turbulence production and Cascade , i) Spectrum of Turbulence in Inertial Sub range, j) Wall-Free Shear Flow—Intermittency, Entrainment, Self-Preservation, Consequence of Self-Preservation in a Plane Jet, Turbulent Kinetic Energy Budget in a Jet; k)Wall-Bounded Shear Flow- inner Layer: Law of the Wall , Outer Layer: Velocity Defect Law , Overlap Layer: Logarithmic Law, Rough Surface, Variation of Turbulent Intensity, l) Eddy Viscosity and Mixing Length, m) Coherent Structures in a Wall Layer, n) Turbulence in a Stratified Medium, o) Taylor's Theory of Turbulent-- *Dispersion,* Rate of Dispersion of a Single Particle, Random Walk, Behavior of a Smoke Plume in the Wind, Effective Diffusivity.

<u>References:</u>

1. Joseph Spurk; **Fluid Mechanics**, Springer (1997).
2. H. Lamb; **Hydrodynamics**, Cambridge University Press (1932).
3. F. Chorlton; **Text book of Fluid Dynamics**; CBS Publicaion, Delhi.
4. S.L. Pai; **Viscous Flow Theory**.
5. F.M. White; **Viscous Fluid Flow**, McGraw-Hill (1974).
6. Milne-Thomson; **Theoretical Hydrodynamics**, Mac Millan & Co.(1949).
7. J.K. Goyel and K.P.Gupta; **Fluid Dynamics**, Pragati Prakashan (1980)
8. H. Schlichting; **Boundary Layer Theory**, McGraw Hill (Senenth Edition, 2014)
9. J.K. Goyel and K.P.Gupta; **Fluid Dynamics**, Pragati Prakashan (1980)
10. J. O. Hinze; **Turbulence**, 2nd ed. (1975), New York: McGraw-Hill.
11. U. Frisch, **Turbulence: The Legacy of A.N. Kolmogorov**, , Cambridge University Press (1995)
12. R. P. Feynman, R. B. Leighton, and M. Sands; **The Feynman Lectures on Physics***,* New York: Addison-Wesley (1963).
13. Pijush K. Kundu and Ira M. Cohen; **Fluid Mechanics**, 3rd Edition, Elsevier (2004).
14. J.H. Ferziger & M. Peric; **Computational Methods for Fluid Dynamics**, Springer, 3rd Edition (2005).
15. C.A.J. Fletcher; **Computational Techniques for Fluid Dynamics**, Vol-I, , Springer, Berlin
16. S. Biringen and C.Y. Chow; **An Introduction to computational Fluid Mechanics by Example**, John Wiley & Sons Inc., Canada (2011)
17. John D. Aderson (Jr.); **Computational Fluid Dynamics- The Basic with Application**, McGraw Hill, Schaum's Outline Series in Mechanical Engineering (1995).