**Semester - 4: Elements of Modern Physics**
**Elements of Modern Physics (Theory)**
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## Davisson and Germer Experiment

Davisson and Germer Experiment, for the first time, proved the wave nature of electrons and verified the de Broglie equation. de Broglie argued the dual nature of matter back in 1924, but it was only later that Davisson and Germer experiment verified the results. They studied the diffraction effects of electron in crystal diffraction experiment using Nickel crystal

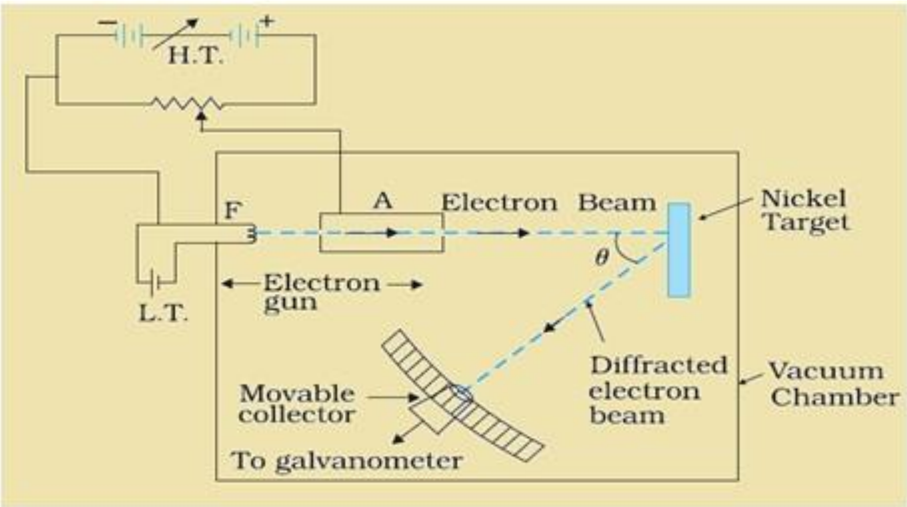



*Fig: The experimental setup.*

The experimental setup for the Davisson and Germer experiment is enclosed within a vacuum chamber to prevent the deflection and scattering of electrons by the medium are prevented. The main parts of the experimental setup are as follows:

- Electron gun: An electron gun is a Tungsten filament that emits electrons via thermionic emission i.e. it emits electrons when heated to a particular temperature.
- Electrostatic particle accelerator: Two opposite charged plates (positive and negative plate) are used to accelerate the electrons at a known potential.
- Collimator: The accelerator is enclosed within a cylinder that has a narrow passage for the electrons along its axis. Its function is to render a narrow and straight (collimated) beam of electrons ready for acceleration.
- Target: The target is a Nickel crystal. The electron beam is fired normally on the Nickel crystal. The crystal is placed such that it can be rotated about a fixed axis.

- Detector: A detector is used to capture the scattered electrons from the Ni crystal. The detector can be moved in a semicircular arc as shown in the diagram above.

In the Davisson and Germer experiment waves were used in place of electrons. These electrons formed a diffraction pattern. The dual nature of matter was thus verified. We can relate the de Broglie equation and the Bragg's law as shown below:

From the de Broglie equation, we have:

$\lambda = h/p = h/\sqrt{2mE}$

$\lambda = h/\sqrt{2mE}$ … (1)

where, m is the mass of an electron, e is the charge on an electron and h is the Plank's constant.

Therefore for a given V, an electron will have a wavelength given by equation (1).

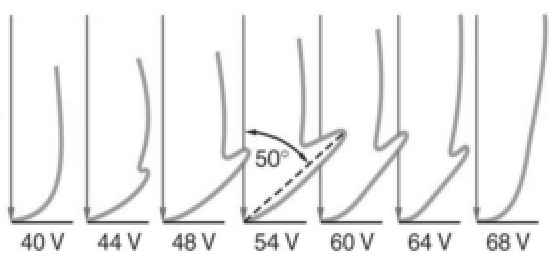
The following equation gives Bragg's Law:

$n\lambda = 2d\sin(90^\circ - \frac{\theta}{2})$ …(2)

Since the value of d was already known from the X-ray diffraction experiments. Hence for various values of θ, we can find the wavelength of the waves producing a diffraction pattern from equation (2).

**Results of the Davisson and Germer Experiment**

- Strong peak was detected at 55V, and the angle of scattering was observed to be 50°
- The pattern of deflected electrons was quite similar to the diffraction pattern of waves
- The wavelength corresponding to the electron (matter wave) was found to be λ = 0.165nm
- The experiment was in strong agreement with De Broglie's hypothesis



*Plots between* I – *the intensity of scattering (X-axis) and the angle of scattering* $\theta$ *for given values of Potential difference.*

Therefore, this establishes the de Broglie's wave-particle duality and verifies his equation as shown below:

From (1), we have: $\lambda = h/\sqrt{2mE}$

For V = 54 V, we have $\lambda = 12.27/\sqrt{54} = 0.167$ nm …. (3)

Now the value of 'd' from X-ray scattering is 0.092 nm.
Therefore for V = 54 V, the angle of scattering is 500,
using this in equation (2), we have:
$n\lambda = 2\ (0.092\ \text{nm}) sin(\ 90^{\circ} - 50^{\circ}/2)$

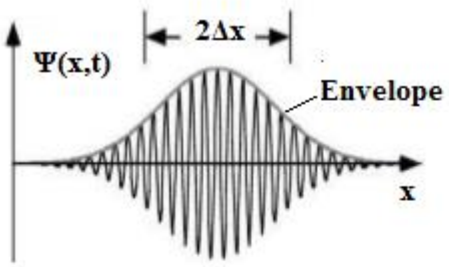
For n = 1, we have: $\lambda = 0.165$ nm ….. (4)

Therefore the experimental results are in a close agreement with the theoretical values got from the de Broglie equation. The equations (3) and (4) verify the de Broglie equation.

**Wave description of particles by wave packets**

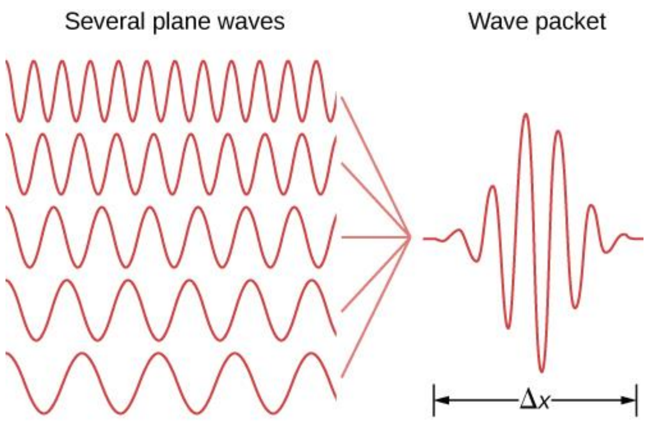
**Wave packets:**
A **wave packet** (or **wave train**) is a short "burst" or "envelope" of localized wave action that travels as a unit.

- Wave packet is a combination of waves with about the same momentum.
- Combining waves into wave packets can provide localization of particles.
- The envelope of the wave packet shows the region where the particle is likely to be found.
- This region propagates with the classical particle velocity.




A wave packet refers to the case where two (or more) waves exist simultaneously. A wave packet is often referred to as a wave group.

This situation is permitted by the principle of superposition. This principle states that if any two waves are a solution to the wave equation then the sum of the waves is also a solution. This principle holds only for linear systems. Following figure explains the situation:



## Group and Phase velocities and relation between them:

We have seen above that waves can be in the group and such groups are called as wave packets, so the velocity with which a wave packet travels is called group velocity. Velocity with which the phase of a wave travels is called phase velocity. The relation between group velocity and phase velocity are proportionate.

### Group Velocity and Phase Velocity

The Group Velocity and Phase Velocity relation can be mathematically written as-

$$V_g = V_p + k\, {}^{dV_p}/_{dk}$$

………(5)

Where,

- $V_g$ is the group velocity.
- $V_p$ is the phase velocity.
- k is the angular wave number.

**Understanding the group and phase velocity:**

Let's first do the superposition of two waves of same amplitude $s_0$

$$s = s_0 \cos(\omega_1 t - k_1 x) + s_0 \cos(\omega_2 t - k_2 x)$$

$$= s_0[\cos(\omega_1 t - k_1 x) + \cos(\omega_2 t - k_2 x)]$$

$$= 2s_0 \left[\cos\left(\frac{(\omega_1 t - k_1 x) - (\omega_2 t - k_2 x)}{2}\right).\cos\left(\frac{(\omega_1 t - k_1 x) + (\omega_2 t - k_2 x)}{2}\right)\right]$$

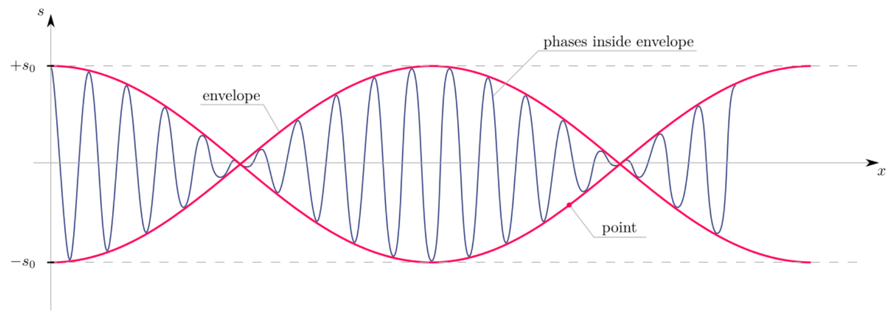
$$= 2s_0 \left[\cos\left(\frac{\omega_1 - \omega_2}{2} t - \frac{k_1 - k_2}{2} x\right).\cos\left(\frac{\omega_1 + \omega_2}{2} t - \frac{k_1 + k_2}{2} x\right)\right]$$

$$= 2s_0 \left[\cos\left(\frac{\Delta\omega}{2} t - \frac{\Delta k}{2} x\right).\cos(\bar{\omega} t - \bar{k} x)\right]$$

…….(6)

Here $\bar{\omega}$ is larger than and $\Delta\omega$ this is why:

- $\cos(\bar{\omega} t - \bar{k} x)$ is a phase part and
- $\cos\left(\frac{\Delta\omega}{2} t - \frac{\Delta k}{2} x\right)$ is amplitude part.




Now, let's consider a point with constant phase defined by, $\bar{\omega}t - \bar{k}x = const.$

Now, velocity of this point will be the phase velocity.

$$\bar{\omega}t - \bar{k}x = const.$$

$$x = \frac{\bar{\omega}}{\bar{k}}t - \frac{const.}{\bar{k}}$$

$$V_p = \frac{dx}{dt} = \frac{\bar{\omega}}{\bar{k}}$$

………(7)

Similarly we can get the Group Velocity **Vg** by keeping the amplitude a constant:

$$\frac{\Delta\omega}{2}t - \frac{\Delta k}{2}x = const.$$

$$x = \frac{\Delta\omega}{\Delta k}t - \frac{const.}{\Delta k}$$

$$V_g = \frac{dx}{dt} = \frac{\Delta\omega}{\Delta k}$$

When $\Delta\omega$ is very small $V_g = \frac{\partial\omega}{\partial k}$

………(8)

***RELATIONSHIP BETWEEN GROUP VELOCITY $V_g$ AND PHASE VELOCITY $V_p$***

Based on the definition **Vp = ω/k**, we can replace **ω** with $k * V_p$, then we get

$$V_g = \frac{\partial\omega}{\partial k} = \frac{\partial(kV_p)}{\partial k} = V_p + k\frac{dV_p}{dk}$$

Since $k = {}^{2\pi}/_{\lambda}$ and $dk = -{}^{2\pi}/_{\lambda^2}\, d\lambda$, then we get

$$V_g = V_p - \lambda\frac{dV_p}{d\lambda}$$

………(9)

*Group Velocity and Phase Velocity relation for Dispersive wave Non-dispersive wave*

| **Type of wave** | **Condition** | **Formula** |
|---|---|---|
| **Dispersive wave** | ${dV_p}/{dk} \neq 0$ | $V_p \neq V_g$ |
| **Non-dispersive wave** | ${dV_p}/{dk} = 0$ | $V_p = V_g$ |

## Probability interpretation: Normalized wave functions as probability amplitudes.

**Wave function**, in quantum mechanics, variable quantity that mathematically describes the wave characteristics of a particle. The value of the wave function of a particle at a given point of space and time is related to the likelihood of the particle's being there at the time. By analogy with waves such as those of sound, a wave function, designated by the Greek letter psi, Ψ, may be thought of as an expression for the amplitude of the particle wave (or de Broglie wave), although for such waves amplitude has no physical significance. The square of the wave function, $\Psi^2$, however, does have physical significance: the probability of finding the particle described by a specific wave function Ψ at a given point and time is proportional to the value of $\Psi^2$.

A **wave function** in quantum physics is a mathematical description of the quantum state of an isolated quantum system. The wave function is a complex-valued probability amplitude, and the probabilities for the possible results of measurements made on the system can be derived from it.

Since wave functions can in general be complex functions, the physical significance cannot be found from the function itself because the $\sqrt{-1}$ is not a property of the physical world. Rather, the physical significance is found in the product of the wave function and its complex conjugate, i.e. the absolute square of the wave function, which also is called the square of the modulus.

$$\Psi^*(r,t)\psi(r,t) = |\Psi(r,t)|^2 \quad \text{………(10)}$$

where *r* is a vector (*x, y, z*) specifying a point in three-dimensional space. The square is used, rather than the modulus itself, just like the intensity of a light wave depends on the square of the electric field.

The **Born** interpretation is that $\Psi^*(r_i)\Psi(r_i)d\tau$ is the *probability* that the electron is in the volume $d\tau$ located at $r_i$. The Born interpretation therefore calls the wave function the probability amplitude, the absolute square of the wave function is called the **probability density**, and the probability density times a volume element in three-dimensional space ($d\tau$) is the probability.

**Normalization of the Wave function:**

A probability is a real number between 0 and 1, inclusive. An outcome of a measurement which has a probability 0 is an impossible outcome, whereas an outcome which has a probability 1 is a certain outcome. According to Equation (10) the probability of a measurement of *x* yielding a result between $-\infty$ and $-\infty$ is

$$P_{x\in-\infty:\infty}(t) = \int_{-\infty}^{+\infty} |\Psi(x,t)|^2 dx$$

However, a measurement of *x must* yield a value between $-\infty$ and $-\infty$, since the particle has to be located somewhere. It follows that

$$P_{x\in-\infty:\infty}(t) = 1$$

or

$$\int_{-\infty}^{+\infty} |\Psi(x,t)|^2 dx = 1$$

which is generally known as the **normalization condition** for the wave function.