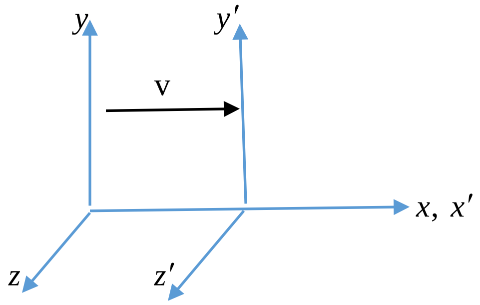
I have tried to develop the following material in a similar way as I deliver my lectures in the Classroom

You will be lost if you just read the study material casually. It is suggestive to go through the arguments a number of times; draw the diagrams, write down every mathematical step, reach the results and observe the consistency of the results with physical arguments.

STUDY MATERIAL (Sem – 4, STR) Dated: 30/03/2020

Already Covered:

- Newtonian Relativity and inadequacy of Galilean Transformation
- Michelson – Morley Experiment and interpretation of Null result
- Propagation of e. m. wave through a moving refracting medium: Fizeau Experiment
- Postulates of Special Theory of Relativity
- Lorentz Transformation and consequences
- Velocity Addition:



Standard relations:

$$u_x = \frac{u'_x + v}{1 + \frac{u'_x v}{c^2}}, \quad u_y = \frac{u'_y\sqrt{1-\beta^2}}{1 + \frac{u'_x v}{c^2}} \quad u_z = \frac{u'_z\sqrt{1-\beta^2}}{1 + \frac{u'_x v}{c^2}}, \quad \beta = \frac{v}{c} \quad \ldots\ldots\ldots\ldots\ldots \quad (1)$$

Tips:

To solve problems of STR, you have to be very careful about physical explanation; only mathematical steps with correct final results without physical explanation can secure only 50% full marks.

Problem:

Show that a 4-dimensioal elementary volume element remains invariant under L.T.

Ans: Let the inertial S'-frame moves with respect to another inertial S-frame along the common $x - x'$ axes with uniform relative velocity v.

Let the volume element $dxdydzdt$ as observed from the S-frame is at rest in the in the S-frame.

(You may also state that $dxdydzdt$ as observed from the S-frame is at rest in the in the S'-frame. In that case the arguments in the following steps will be different).

Then with respect to the S'- frame, the volume element is moving along the negative $x-x'$ with velocity v . With respect to the S'-frame, the volume element is $dx'dy'dz'dt'$.

Now clearly $dx' = dx\sqrt{1-\beta^2}$ when $\beta = \frac{v}{c}$. Also $dt' = \frac{dt}{\sqrt{1-\beta^2}}$ The rest part of the problem is clear.

NOTE:

If you assume the part within the bracket above, then try to set up the steps and arguments.

Dynamics and Relativity:

We now try to understand the dynamics in Relativistic frame. In non-relativistic Newtonian mechanics, let a body A applies a force say, on a body B. Now the situation may be a collision (elastic or inelastic); in this case, the bodies exchange force when A and B are in contact. Another possibility may be application of force at a distance ("action at a distance" force). In the second case, the bodies A and B are separated in space when they exchange force. This second case should be understood very carefully. We know that action and reaction are **Simultaneous**, but in relativity **Simultaneity** is a relative concept. Since body A and body B are separated in space, then how do you establish the simultaneity of action and reaction in different inertial frames. So "action at a distance" has no meaning in relativity.

(**Students, please try to make you satisfied with the argument given in the previous paragraph, its very important if we want to proceed further)**

So, we now try to understand the exchange of force during elastic collision in which the **separation between the bodies is negligibly small and the time during which the forces act is infinitesimal.**

Let us consider two inertial frames S and S' which are moving along their common $x-x'$ axes with uniform velocity v. We have two balls A and B of identical mass which are moving in the common $x-y$ plane or common $x'-y'$ plane and cause elastic collision. We will observe the physical aspects before and after collision (see the diagram below).

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<u>OBSERVATION FROM S' - FRAME</u>:

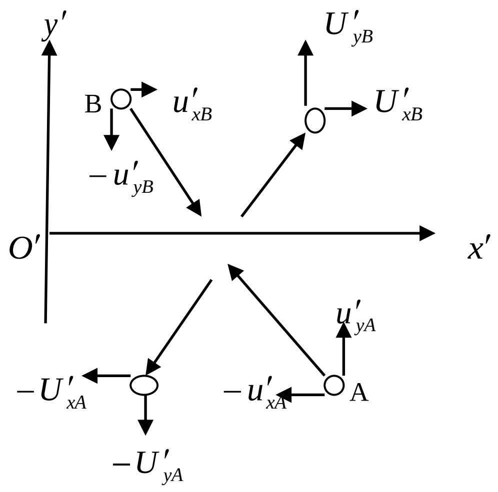

Fig. 1

We will use **small letters for description before collision** and **capital letters after collision**. We have introduced above a situation in which the motion before collision is highly symmetrical in the $x' - y'$ plane, i.e. two bodies of identical mass approaches each other with same velocity.

<u>BEFORE COLLISION:</u>

From symmetry, we can write

$$u'_{xB} = -u'_{xA} \quad \text{and} \quad -u'_{yB} = u'_{yA} \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

(Check from the diagram above, the –ve sign is obvious).

<u>AFTER COLLISION:</u>

$$U'_{xB} = u'_{xB} = -U'_{xA} = -u'_{xA} \quad \text{and} \quad U'_{yB} = -u'_{yB} = -U'_{yA} = u'_{yA} \quad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

(Check from the diagram above)

<u>NOTE:</u> The y-components of velocity are simply reversing their signs and the x-components remain unchanged. (See equations (2) and (3) and also the diagram).

Without loss of generality we are now taking an assumption which will make the dynamics simpler. Let $v = u'_{xB} = -u'_{xA}$.

What is the result of this assumption? Try to understand.

v is the velocity of the S'- frame with respect to the S-frame along the common $x - x'$ axes. Now if $v = -u'_{xA}$ = the x-component of velocity of body A, then the S-frame observer will see that the body A has no x-component of velocity and it moves along the y-direction (diagram below). [Satisfy yourself with this argument before you proceed further]

OBSERVATION FROM S - FRAME:

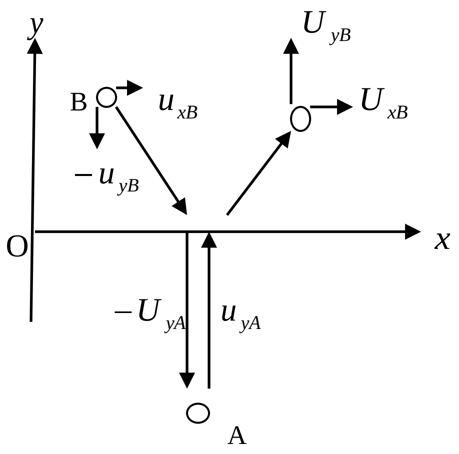

Fig. 2

We are interested to investigate whether the conservation of linear momentum remains unchanged in both frames.

In non-relativistic Newtonian mechanics, the y-component of velocity is not affected under transformation from one inertial frame to another frame.

Hence for the bodies A and B, we get following the above argument and fig. 1 and fig. 2

$$u_{yA} = u'_{yA} = -U_{yA}, \quad -u_{yB} = -u'_{yB} = -U_{yB} \quad \text{……………….. (4)}$$

Thus in the S-frame, the conservation of linear momentum demands:

Momentum lost by A = Momentum gained by B

$$2mu_{yA} = 2mu_{yB} \Longrightarrow u_{yA} = u_{yB} \quad \text{………… (5)}$$

What will happen if we approach from relativistic point of view?

From equation (1)

$$u'_{yB} = \frac{u_{yB}\sqrt{1-\beta^2}}{1-\frac{u_{xB}v}{c^2}} \quad \text{…………….. (6)}$$

$$u'_{yA} = u_{yA}\sqrt{1-\beta^2} \qquad as\ u_{xA} = 0 \text{ (fig. 2) ………… (7)}$$

From eqn (6) and (7), note that if the y-components of velocity are same in one frame (say, if $u'_{yA} = u'_{yB}$) then they are not necessarily equal in the other frame.

Thus for $u'_{yA} = u'_{yB}$: $$u_{yA} = \frac{u_{yB}}{1-\frac{u_{xB}v}{c^2}} \qquad \text{............................} \quad (8)$$

And this contradicts Newtonian result (non-relativistic): eqn (5).

Observation: If we define $\vec{p} = m\vec{u}$ in S-frame and $\vec{p}' = m\vec{u}'$ in the S' - frame, then if it is conserved in one frame, it is not conserved in the other frame. But this contradicts the postulate of relativity.

IT IS NOW NECESSARY TO REDINE LINEAR MOMENTUM FROM RELATIVISTIC POINT OF VIEW.

Linear Momentum:

Try to understand the fact that the equation (5) results because of the assumption of equal mass of both the bodies A and B. We are now trying to redefine linear momentum with the notion that mass of a body may depend on its motion, i.e. we are attempting to observe whether motion affects mass (if yes: how?).

Let masses of A and B are identical when those are at rest (and let it is m_0). Then in motion, equation (5) becomes (in relativistic point of view)

$$2m_A u_{yA} = 2m_B u_{yB} \qquad \text{...........................} \ (9)$$

When m_A and m_B are masses of A and B when in motion.

Hence $$m_B = m_A \frac{u_{yA}}{u_{yB}} = \frac{m_A}{1-\frac{u_{xB}v}{c^2}} \quad \text{(using eqn (8) ....)} \quad (10)$$

Now we simplify the above equation with the assumption $v = u'_{xB}$ (see Page 3: below equation (3)), i. e.

$$u'_{xB}(=v) = \frac{u_{xB} - v}{1-\frac{u_{xB}v}{c^2}} \qquad \text{.................................} \ (11)$$

Home Work: Solve equation (11) and get $$v = \frac{c^2}{u_{xB}}\left[1-\sqrt{1-\left(\frac{u_{xB}}{c^2}\right)^2}\right] \qquad \text{........} \ (12)$$

Now use eqn (12) ineqn (10) and get

$$m_B = \frac{m_A}{\sqrt{1-\left(\frac{u_{xB}}{c}\right)^2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (13)$$

Now if we take the body A is at rest in the S-frame with mass $m_A = m_0$ and m be the mass of the body B which moves with velocity u_{xB}, then

$$m = \frac{m_0}{\sqrt{1-\left(\frac{u_{xB}}{c}\right)^2}}$$

When u_{xB} is the velocity of B along the x axis in the S-frame. If we put C, then

$$m = \frac{m_0}{\sqrt{1-\left(\frac{u}{c}\right)^2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.. \quad (14)$$

As $\frac{u}{c} \to 0$, $m \to m_0$.