Sometimes, a population can be divided into 2 groups, each member of one group containing an attribute A, and members of the other group, not containing A. Let N_1 be the no. of units in the population containing A. ~~We are~~. Our objective is to estimate the population proportion $P = \frac{N_1}{N}$, by SRS.

Let n_1 be the no. of units in the sample possessing A. Hence, $p = \frac{n_1}{n}$ is the sample proportion.

Let y be the r.v $\ni$ its value on one unit i is Y_i, i.e. $Y_i = 1\ (0)$ if i contains A (otherwise).

$$\therefore \sum_{i=1}^{N} Y_i = Y \text{ (, let)} = N_1 \quad , \quad P = \bar{Y}.$$

$$\sum_i y_i = n_1 \ , \quad p = \frac{1}{n}\sum_{i=1}^{n} y_i = \bar{y}.$$

Also, $\sum_i Y_i^2 = N_1 = NP$. $\quad \sum_i y_i^2 = n_1 = np$.

<u>SRSWR.</u>

$\because$ a sample mean is an UE of population mean

$$E(p) = P.$$

The estimated no. of units in the population with A is $\hat{N}_1 = \hat{Y} = Np$.

Again, $V(p) = V(\bar{y}) = \frac{1}{n} \cdot \frac{1}{N} \sum_i (Y_i - \bar{Y})^2$

$$= \frac{1}{n} \cdot \frac{1}{N} \left(\sum_i Y_i^2 - N\bar{Y}^2 \right).$$

$$= \frac{1}{nN}(NP - NP^2) = \frac{PQ}{n} \ , \quad Q = 1 - P$$

An UE of $V(p)$ is

$$v(p) = v(\bar{y}) = \frac{1}{n} \cdot \frac{1}{n-1} \sum_i (y_i - \bar{y})^2$$

$$= \frac{1}{n(n-1)} \left(\sum_i y_i^2 - n\bar{y}^2 \right) = \frac{1}{n(n-1)} (np - np^2).$$

$$= \frac{pq}{n-1}, \quad q = 1-p.$$

It can be easily verified that $E(v(p)) = V(p)$.
*

SRSWOR

$\because E(\bar{y}) = \bar{Y}$, p is an UE of P.

$$V(p) = V(\bar{y}) = \frac{N-n}{Nn} S_y^2 = \frac{N-n}{(N-1)Nn} \sum_i (Y_i - \bar{Y})^2$$

$$= \frac{N-n}{Nn(N-1)} \left[\sum_i Y_i^2 - N\bar{Y}^2 \right] = \frac{N-n}{Nn(N-1)} \left[NP - NP^2 \right]$$

$$= \frac{N-n}{n(N-1)} PQ.$$

An UE of $V(p)$ is

$$v(p) = v(\bar{y}) = \frac{N-n}{Nn} s_y^2 = \frac{N-n}{Nn(n-1)} \sum_i (y_i - \bar{y})^2.$$

$$= \frac{N-n}{Nn(n-1)} (np - np^2) = \frac{N-n}{N(n-1)} pq.$$

It can be verified that $E(v(p)) = V(p)$.
**

NB: * and ** is left as an exercise to the reade