

STATISTICS-Sem 2-Gen-Bayes' Theorem-Saptarshi Mondal-31 Mar 2020

What is the Bayes' Theorem?

Bayes' theorem, named after 18th-century British mathematician Thomas Bayes, is a mathematical formula for determining conditional probability. The theorem provides a way to revise existing predictions or theories (update probabilities) given new or additional evidence.

Bayes' theorem is also called Bayes' Rule or Bayes' Law and is the foundation of the field of Bayesian statistics.

The Formula For Bayes' Theorem Is

$$P(A|B) = P(A \cap B) / P(B) = P(A) \cdot P(B|A) / P(B)$$

where: $P(A)$ = The probability of A occurring

$P(B)$ = The probability of B occurring

$P(A|B)$ = The probability of A given B

$P(B|A)$ = The probability of B given A

$P(A \cap B)$ = The probability of both A and B occurring

Example: In a certain factory, machines A, B, and C are all producing springs of the same length. Of their production, machines A, B, and C produce 2%, 1%, and 3% defective springs, respectively. Of the total production of springs in the factory, machine A produces 35%, machine B produces 25%, and machine C produces 40%. Then we have

$$P(D|A) = 0.02, P(A) = 0.35;$$

$$P(D|B) = 0.01, P(B) = 0.25;$$

$$P(D|C) = 0.03, P(C) = 0.40.$$

If one spring is selected at random from the total springs produced in a day, the probability that it is defective equals

$$P(D) = \sum P(D|X)P(X) = 215/10000$$

If the selected spring is defective, the conditional probability that it was produced by machine A, B, or C can be calculated by

$$P(A|D) = P(D|A)P(A)/P(D) = 70/215$$

$$P(B|D) = P(D|B)P(B)/P(D) = 25/215$$

$$P(C|D) = P(D|C)P(C)/P(D) = 120/215$$